

**Math Review to the paper** *M. Atapour, C. E. Soteris, D. W. Sumners and S. G. Whittington, ‘Counting closed 2-manifolds in tubes in hypercubic lattices’, Journal of Physics A: Mathematical and Theoretical, 48 (2015).*

This is a paper on pure mathematics (geometry and topology) motivated by physics. The authors study asymptotic behavior of closed connected 2-manifolds in the union of 2-dimensional faces of the standard cubic lattice in  $d$ -dimensional Euclidean space  $R^d$ .

E.g. for each  $d \geq 3$  and  $L$  there are  $\alpha, \lambda > 0$  such that the number of polyhedral spheres

- formed by  $n$  two-dimensional faces of the standard cubic lattice in  $R^d$ ,
- whose vertices satisfy inequalities  $x_1 \geq 0, \quad 0 \leq x_2 \leq L, \quad \dots, \quad 0 \leq x_d \leq L,$
- who have a vertex for which  $x_1 = 0,$

is asymptotically equivalent to  $\alpha \lambda^n$  (Theorem 4).

Same result (with different  $\alpha, \lambda$ ) holds for ‘polyhedral spheres’ replaced by ‘closed connected 2-dimensional manifolds’ (Theorem 5).

The authors also show that

- manifolds with any fixed genus,
- orientable manifolds for  $d \geq 4$ , and
- unknotted manifolds

are exponentially rare (Theorems 6-9).

The statements and proofs are not up to mathematical standards, so there appears an interesting task of writing rigorous statements and proofs.