Dynasty Fellowship Report 2011

Valentina Kiritchenko

1. Main results

In 2011, my main results are a construction of convex geometric divided difference operators (joint with Evgeny Smirnov and Vladlen Timorin) and a description of equivariant algebraic cobordism of complete flag varieties and of minimal rank wonderful symmetric varieties (joint with Amalendu Krishna). I also continued the joint project with Klaus Altmann and Lars Petersen on a relation between colored fans on one side and polyhedral divisors on wonderful compactifications on the other side.

Divided difference operators (or Demazure operators) play a key role in Schubert calculus and representation theory. We constructed convex geometric analogs of Demazure operators. Geometric Demazure operators act on polytopes and take a polytope to a polytope of dimension one greater. For instance, Gelfand-Zetlin polytopes can be obtained by applying a suitable composition of geometric Demazure operators to a point. In contrast with the classical Demazure operators, the convex geometric Demazure operators can be defined not only for the root system of a reductive group but for a more general combinatorial datum. The construction and main results are outlined in [KST3]. This can be further developed in three directions. First, the geometric Demazure operators can be used to give a convenient description of Newton–Okounkov polytopes for Bott-Samelson resolutions of Schubert varieties (for GL_n such a description can be deduced from the iterative construction of these polytopes given by Dave Anderson). In particular, an elementary description of *string polytopes* (generalizations of Gelfand–Zetlin polytopes to arbitrary refductive groups) can be obtained this way. Second, the results of [KST1] (presentation of Schubert cycles by faces of the Gelfand–Zetlin polytopes and formulas for the Demazure characters via exponential sums over integer points in these faces) can be extended to arbitrary reductive groups. In particular, a general version of *mitosis* (a combinatorial procedure for computing Schubert polynomials in the case of GL_n in terms of pipe-dreams introduced by Knutson and Miller) can be constructed this way. Third, the geometric Demazure operators can be studied in a more general setting (not related to the classical root systems). For instance, Newton polytopes of *Bott towers* (toric varieties obtained from a point by successive projectivizations of rank two vector bundles) admit a simple description via geometric Demazure operators.

Equivariant algebraic cobordism is defined for algebraic varieties (over a field k of zero characteristic) with an action of an algebraic group G. The definition reminds of the definition of equivariant Chow rings, namely, instead of considering the universal classifying space BG (which is not algebraic) one takes a sequence of algebraic varieties approximating BG and then takes the inverse limit of their Chow (or cobordism) rings. The only difference is that for cobordisms this sequence should be chosen more carefully (since cobordism can be non-zero for arbitrary negative degrees). The resulting equivariant cobordism ring can be non-trivial in arbitrary positive as well as negative degrees. For

instance, if G is a torus of dimension n then the equivariant cobordism ring of a point is isomorphic to the ring of *graded* power series in n variables with coeffients in the Lazard ring (that is, coincides with the complex cobordism ring of BG). For a connected reductive group G split over k, we obtained a Borel type presentation for the equivariant algebraic as well as complex cobordism of the variety of complete flags G/B. We also computed the equivariant algebraic cobordism ring for the symmetric varieties of minimal rank by extending the methods of Brion and Joshua from equivariant Chow rings to cobordisms.

We relate the language of *colored fans* (combinatorial objects describing spherical varieties) and the language of *polyhedral divisors* (describing varieties with a torus action). Colored fans are usual fans together with an additional combinatorial data (colors), namely, some rays in the fan can be colored. Polyhedral divisors are linear combinations of usual divisors with coefficients being polyhedral fans (that is, decompositions of an affine space into polyhedra). Each spherical variety can be described by a colored fan as well as by a polyhedral divisor on a suitable *wonderful compactification* (the latter are equivariant compactifications of homogeneous spaces with nice properties, in particular, they have a unique closed orbit). Our plan is to describe the polyhedral divisor in terms of the colored fan and vice versa. Previously, we did this for horospherical varieties but recently realized that the same approach works for arbitrary spherical varieties.

2. Publications and preprints

[KST1] joint with Evgeny Smirnov and Vladlen Timorin, *Schubert calculus and Gelfand-Zetlin polytopes*, 33 pages, arXiv:1101.0278v2 [math.AG]

[KST2] joint with Evgeny Smirnov and Vladlen Timorin, *Gelfand-Zetlin polytopes and Demazure characters*, Proceedings of the International Conference "50 years of IITP", 5 pages, Moscow, IITP RAS, 2011

[KST3] joint with with Evgeny Smirnov and Vladlen Timorin, *Convex chains for Schubert varieties*, Oberwolfach reports, 41/2011, 15-18

[KK] joint with Amalendu Krishna, *Equivariant Cobordism of Flag Varieties and of Symmetric Varieties*,18 pages, arXiv:1104.1089v1 [math.AG]

[HK] joint with Jens Hornbostel, *Schubert calculus for algebraic cobordism*, Journal für die reine und angewandte Mathematik (Crelle), Volume 2011, no. 656, 59–85

3. Talks

Conference talks

July	International Conference "50 years of IITP", Moscow
August	Oberwolfach mini-workshop "New developments in Newton–Okounkov bodies", Oberwolfach, Germany

Seminar talks

February	Seminar of the Laboratory of algebraic geometry, Higher School of Economics, Moscow

April	Seminar Riemann surfaces, Lie algebras and Mathematical physics, Independent University of Moscow
November	Lie groups and Invariant theory Seminar, Moscow State University

4. International collaboration

Freie Universität Berlin	project "Spherical varieties and polyhedral divisors on wonderful compactifications", joint with Klaus Altmann and Lars Petersen (visited Moscow in November 2011)
Tata Institute, Mumbai	project "Equivariant cobordism of spherical varieties", joint with Amalendu Krishna

5. Teaching

I teach on a regular basis at the Faculty of Mathematics, Higher School of Economics. Together with Alexander Kolesnikov, I teach a course "Calculus of variations and optimal control":

`http://www.hse.ru/edu/courses/34463194.html`

I participate in problem solving sessions for the 1st year undergraduate students: courses Calculus I, Algebra I, Geometry I and Logic. Together with Alexey Gorodentsev, I run an undegraduate learning seminar "Toric geometry, Grassmannians, flags and symmetric functions".

`http://vyshka.math.ru/1112/sem_gor-kir.html`

In February 2011, I gave a lecture "Polytopes and equations" at the winter mathematical school for university students organized by the HSE:

`http://www.hse.ru/news/recent/26949846.html`

In summer 2011, I taught a course "Schubert calculus and 3264 conics" for high school students at the summer school "Contemporary mathematics in Dubna":

`http://www.mccme.ru/dubna/2011/courses/kirich.htm`

I supervise a 4th year student (Diploma "f-vectors of Gelfand–Zetlin polytopes") and two 2d year students (course projects "Schubert polynomials and Fomin–Kirillov theorem").

At the HSE, I coordinate the PhD program in Mathematics. In particular, I was responsible for the admission exams and for licensing of a new PhD program in Geometry and Topology in 2011. I've also prepared a proposal for an "academic" PhD program in Mathematics (the "academic" or "full-time" PhD program already exists at the HSE for some sciences and provides scholarships worth 25000 Rubles per month).