Nikolay Gusev: Summary

In many cases mathematical treatment of liquids is done in the framework of *incompressible* fluid. However, from the physical point of view, all the liquids existing in nature are *low compressible.* Therefore it is reasonable to study the equations of low compressible fluid motion, in particular, convergence of their solutions to the corresponding incompressible limit.

To emphasize the fact that the fluid is low compressible one should introduce a parameter which represents a measure of *compressibility.* Since the density of the incompressible fluid is constant, it is natural to define the *compressibility* as the *deviation* of the density from this constant. This approach was proposed by E.G. Shifrin, who suggested an equation of state with *bounded* density. The advantage of this approach in comparison with other approaches (used by D.G. Ebin, S. Klainerman, P.L. Lions, N. Masmoudi, E. Feireisl and other authors) is that the classical homogeneous incompressible Navier–Stokes system is immediately and *explicitly* obtained when the compressibility is zero.

Another interesting property of the equation of state suggested by E.G. Shifrin is that the density is clearly *a priori* bounded from below and above when the compressibility is sufficiently small. Usually the derivation of estimates of density is much more involved and is one of key steps in the proof of existence of weak solution to compressible Navier–Stokes equations.

In a recent preprint "Asymptotic Properties of Linearized Equations of Low Compressible Fluid Motion" the author considered the simplest case of linearized Navier–Stokes equations with linear equation of state. Existence & uniqueness of weak solutions were established and convergence of these solutions as the compressibility tends to zero was studied. The goal of this project is to develop analogues of some of these results for the original nonlinear Navier–Stokes equations with the equation of state suggested by E.G. Shifrin. In particular, the following questions are to be addressed:

1. Existence of weak solutions to initial–boundary value problem in dimensions 2 and 3.

2. The behavior of the weak solutions when the compressibility tends to zero.

3. Regularity and uniqueness of the solutions in dimension 2.