DYNASTY FOUNDATION COMPETITION

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Summary.

Ergodic theory, together with its connections to representation theory, group theory and Teichmüller theory, is the subject of this proposal. The proposal is articulated in four sections:

1) **The Vershik–Kerov Conjecture.**

In 1985 Vershik and Kerov conjectured that suitability normalized dimensions of irreducible representation of the symmetric group converge to a constant according to the Plancherel measure.

In 2010 I proved the Vershik–Kerov Conjecture (paper accepted to GAFA).

The next step is to study limit theorems for local patterns in Young diagrams. Very interesting progress in this direction has recently been made by K. Tolmachov.

2) **Finiteness of ergodic measures.**

In 2011 I showed that ergodic unitarily invariant measures defined on the space of infinite Hermitian matrices and admitting well-defined projections on some "finite"corner, must be *finite*. It would be interesting to extend this result to other series of infinite-dimensional homogeneous spaces.

3) **Limit theorems for translation flows.**

Ergodic integrals of translation flows on flat surfaces do not converge in distribution. I have obtained a limit theorem for translation flows: it turns out that the distributions of normalized ergodic integrals converge to an orbit if an ergodic dynamical system in the space of probability measures.

The next step is to study smoothness properties of accumulation points of distributions of ergodic integrals. Another direction, proposed by A. Katook, is to understand what happens for smooth area–preserving flows on surfaces.

4) **Ergodic theorems for group actions.**

In a joint paper with M. Yu. Khristoforov and A. V. Klimenko (accepted to IMRN), Cesàro convergence is established for spherical averages corresponding to measure–preserving action of Gromov hyperbolic groups, and, more generally, Markov groups. The limit is not known to be invariant under the action, however, — this is the next question. We also plan to extend our result from bounded to Lebesgue–integrable functions.

A harder question is to investigate convergence of spherical averages themselves, rather than that of their Cesàro means.