Topological methods in convex and discrete geometry
Short summary
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The topological methods in discrete and convex geometry have been used around 100 years, starting from the work of Brouwer, Sperner, Borsuk. Other classical examples are the "ham sandwich theorem" of Stone–Tukey–Steinhaus and the central point of Neumann–Rado.

More recent development are connected with the Tverberg theorem, which is a deep generalization of the central point theorem. Bárány, Shlosman, Szűcs started the application of topological methods to the Tverberg theorem, later strengthened in the work of Živalević, Vrećica, Blagojević, Matschke, Ziegler to give different versions of the "colorful" Tverberg theorem.

Boros, Füredi, and Bárány proved another result in the spirit of the central point theorem: for any finite set $X \in \mathbb{R}^n$ find a point x such that the probability of covering x by a random simplex with vertices in X is at least some constant c_n. The proof in arbitrary dimension used the Tverberg theorem. Recently Gromov applied a topological method was applied to this problem, giving the best known constant c_n. After that in my short paper the proof of Gromov was greatly simplified.

Another area of research are inscription theorems going back to the Schnirelmann theorem on inscribed squares. In the work of Makeev and mine the Schnirelmann theorem was generalized to inscribing crosspolytopes (higher-dimensional octahedra).

The same topological facts are used in different measure partition theorems. In my recent paper several theorems on measure partition were deduced from the result about configuration spaces, deeply generalizing the ham sandwich theorem.

Topological methods were used by Dol'nikov and me in the positive solution of the Gromov–Milman conjecture about the algebraic analogue of the Dvoretzky theorem.

Another active field of research is the study of guaranteed complexity of preimages of points in different senses. Typical examples are: the Alexandrov and Urysohn width, the Gromov waist, the Lusternik-Schnirelmann category or the Schwarz genus of preimages, and the cardinality of a preimage. The Boros–Füredi–Bárány–Gromov theorem is also a result about multiplicity.

Most of the above results use certain facts from the topology of configuration spaces (of q-tuples of point), which may be stated as the general direction of the research.

The following particular question are addressed in the planned research: Complexity of preimages. For example, Gromov's theorem on the waist of the sphere is extended to the case of arbitrary manifold Y as the codomain space. We also try to replace the domain space X by Riemannian manifolds other than the round sphere in theorems on the Urysohn width and the Gromov waist. Some geometric applications of the lower bounds of multiplicity of a continuous map will be examined.

Extensions of the Rattray theorem about "nonlinear orthogonalization" and their corresponding measure partition theorems in the spirit of V.V. Makeev. Some questions about the topology of configuration spaces are also studied. Products of configuration spaces with application to some geometric problems, in particular to geometry of normed spaces. Approaches to the Bang conjecture about covering of a convex body by planks along with possible generalizations of the Kadets theorem about the sum of inradii to arbitrary norms.

Some questions on the topology of configuration spaces important in robotics and motion planning.

Topological and algebraic methods in additive combinatorics of finite fields.