Summary

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The moduli space of a certain class of geometric objects parametrizes the isomorphism classes of these objects. These kind of spaces are studied very intensively during the last several decades both because of their rich internal geometric and topological structure and because of their close connections with physics. My research focuses on two types of moduli spaces: the moduli space of algebraic curves and the moduli space of framed torsion free sheaves on the projective plane.

The moduli space of curves is the space of all possible complex structures on a given 2-dimensional surface of a fixed topological type. The interest in the moduli space of curves is inspired by applications in topological string theory and, from mathematical side, Gromov-Witten theory – the modern way to count algebraic curves in a fixed target variety. In the last 20 years it was observed that Gromov-Witten theory is strongly related to the theory of integrable hierarchies of PDEs of hydrodynamic type. On one side this connection allows to get a deep understanding of a structure of Gromov-Witten invariants and on the other side it gives a clue for a proof of important results about integrable hierarchies.

The topology of the moduli space of sheaves is endowed with different types of algebraic structures which are important from the point of view of representation theory. The moduli space of sheaves is also a source of a large family of very interesting spaces – Nakajima's quiver varieties. They are examples of symplectic resolutions and are very important for algebraic geometers. Different combinatorial structures are hidden in the moduli space of sheaves. The topology of the moduli space of sheaves provides an understanding of different results in partition theory and in the theory of symmetric functions.

One of the main problems in the topology of the moduli space of curves is a description of a certain subalgebra in the cohomology algebra – the so-called tautological ring (see [Fab]). This ring is one of the main objects in my research. In the joint work with S. Shadrin ([BS2]) we gave a new proof of Faber's conjecture about top intersections in this ring. I have two works in progress now: joint work with S. Shadrin, L. Spietz and D. Zvonkine about certain cycles in the moduli space of curves – double ramification cycles; a proof of Zvonkine's conjecture ([Zvon]) about the top tautological group of a certain version of the moduli space of curves. There are a lot of conjectures about the tautological ring. This is the direction of my future research. Relations in the tautological ring automatically give relations between Gromov-Witten invariants. My goal is to apply this techniques for a proof of the Virasoro conjecture in Gromov-Witten theory (see [Pan]). Dubrovin and Zhang ([DZ]) constructed a correspondence between Gromov-Witten theory and integrable hierarchies. There is an unproved part in their construction, that they left as a conjecture. In my joint works with H. Posthuma and S. Shadrin ([BPS1, BPS2]) we proved a half of this conjecture. Proving the remaining part of this conjecture is a goal of my future research. My approach is partially based on certain deformation formulas that we found in the joint work with S. Shadrin ([BS1]).

The group $GL_2(\mathbb{C})$ acts on the projective plane, this action lifts to the action on the moduli space of sheaves. H. Nakajima (see [Nak]) developed a rich theory about varieties that are fixed points of actions of finite subgroups of $SL_2(\mathbb{C})$ on the moduli space of sheaves. My goal is to generalize this theory for subgroups of $GL_2(\mathbb{C})$ that doesn't belong to $SL_2(\mathbb{C})$ and study applications in combinatorics. In my work [B1] and joint works [BF1, BF2] I obtained several results in the case of a one-dimensional subtorus in $GL_2(\mathbb{C})$. The other goal is study applications in combinatorics, in particular in the theory of plane partitions. I made a progress in this direction in my work [B2].

There are several papers that I wrote during my undergraduate studies and that are not related to my research interests now (see [B3, B4, B5, B6]).

Literature

Publications of A. Buryak related to the topics of the project:

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