Summary of the research statement

"Geometry of hypercomplex manifolds"

Andrey Soldatenkov

The research project is devoted to the study of the geometry of hypercomplex manifolds. A smooth manifold is called hypercomplex if it is equipped with a triple of integrable almost-complex structures satisfying quaternionic relations. There exist many examples of such manifolds including hyperkähler manifolds, nilmanifolds, Lie groups with hypercomplex structures and others. Homogeneous hypercomplex structures on Lie groups appeared in the context of string theory and then in the work of D. Joyce. Hypercomplex manifolds are interesting both from the point of view of differential geometry and of the complex algebraic geometry.

Each hypercomplex manifold is endowed with a torsion-free connection preserving all of the complex structures. This connection is known as the Obata connection. The holonomy group of this connection is contained in $GL(n, \mathbb{H})$, which is one of the groups in the list of possible irreducible holonomies. The classification of irreducible holonomy groups of torsion-free connections, initiated by Cartan and Berger, was completed in 1999 by Merkulov and Schwachhöfer. As it was shown in [1], the Obata connection on the Lie group $SU(3)$ with left-invariant hypercomplex structure has irreducible holonomy and the holonomy group coincides with $GL(2, \mathbb{H})$. This provides the first known example of a connection with such holonomy on a compact manifold.

The tangent bundle of each hypercomplex manifold is endowed with an action of the quaternion algebra. Every purely imaginary unitary quaternion provides an integrable almost-complex structure called the induced complex structure. Thus the hypercomplex manifold is endowed with a family of complex structures parametrized by the points of two-dimensional sphere. Some properties of this family were studied in [2]. It was shown that under some additional assumptions the complex manifold corresponding to the generic induced complex structure contains no divisors, and all complex subvarieties of codimension two are trianalytic (that is, analytic with respect to all of the complex structures).

The possible directions of future research include:

- Further study of the homogeneous hypercomplex structures on Lie groups.
- The study of subvarieties of hypercomplex manifolds, especially holomorphic Lagrangian subvarieties and holomorphic Lagrangian fibrations.
- The study of special metrics on hypercomplex manifolds and the quaternionic analogue of the Calabi conjecture.

[1] A. Soldatenkov, *Holonomy of the Obata connection on* $SU(3)$, Int. Math. Res. Notices (2012), Vol. 2012 (15), 3483-3497.

[2] A. Soldatenkov, M. Verbitsky, *Subvarieties of hypercomplex manifolds with holonomy in* $SL(n, \mathbb{H})$, J. Geom. Phys. (2012), Vol. 62 (11), 2234-2240.