Hochschild-Witt cohomology and a cristalline analog of the Tate Conjecture

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Summary

We define a non-commutative analog of the de Rham-Witt complex and cristalline cohomology for arbitrary associative unital algebras over a finite field. We generalize this to DG algebras, so that in particular, we can recover the usual cristalline cohomology and de Rham complex of smooth algebraic varieties (in the same sense as peridoci cyclic homology recovers de Rham cohomology). We construct a regulator map from algebraic K-theory to appropriate weighted cohomology groups constructed from our non-commutative cristalline cohomology, and we prove that for a finite-dimensional algebra A over an algebraic closure of a finite field, the regulator map becomes an isomorphism after pro-p completion. Finally, we try to extend this statement to Hom-finite DG algebras and D categories, so that it applies to smooth proper algebraic varieties and gives a weak version of the cristalline analog of the Tate conjecture.