Summary

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The Aims of Research are study of complex geometry on moment-angle manifolds and their partial quotients, study of the mixed Hodge structure on open toric varieties.

The First Part of this research project is devoted to computation of the mixed Hodge structure on open toric varieties and complements to coordinate subspace arrangements. Let Z be an arrangement of coordinate subspaces in $\mathbb{C}^n$. The cohomology ring of $\mathbb{C}^n \setminus Z$ was computed by V. Buchstaber and T. Panov. There is a bigrading on $H^*(\mathbb{C}^n \setminus Z)$ that was introduced from topological reasons. Because $\mathbb{C}^n \setminus Z$ is complex manifold there is the Hodge filtration on $H^*(\mathbb{C}^n \setminus Z)$. We computed this filtration and showed that bigrading from the Hodge filtration coincides with the topological bigrading (2013). We also used this results to construct a family of integral representations of holomorphic functions.

Complements to coordinate subspace arrangements are particular cases of open toric varsities. Cohomology groups of smooth open toric varieties were computed by M. Franz. **The research problem** is to compute the mixed Hodge structure on cohomology of a open toric varsities.

The Second Part and the main part of our research project is devoted to study the complex geometry of moment-angle manifolds and their partial quotients. The general construction of moment-angle manifold was introduced in the toric topology setting see the book of V. Buchstaber and T. Panov, but particular cases were appeared as a part of symplectic reduction construction for toric manifolds. The fact that the moment-angle manifold admits a complex structure was discovered quite recently, in the last decade the series of papers on this topic was published, see papers of L. Meersseman, S. López de Medrano, A. Verjovsky, T. Panov, Yu. Ustinovsky, M. Verbitsky, H. Ishida. Two particular cases of such complex manifolds have been known for a long time, that is the Hopf manifold and the Calabi-Eckmann manifold. The Hopf manifold is diffeomorphic to $S^{2n+1} \times S^1$, and the Calabi-Eckmann manifold is diffeomorphic to $S^{2n+1} \times S^{2m+1}, n > 0, m > 0$. The Calabi-Eckmann and Hopf manifolds are classical examples of non-Kähler manifolds and their complex geometry is well-studied. Complex structures on moment-angle manifolds may be considered as generalizations of Calabi-Eckmann and Hopf manifolds.

Let $\mathcal{Z}_{\mathcal{K}}$ be a moment-angle manifold with a complex structure. **The problem** is to compute the Dolbeault cohomology and cohomology of different sheafs. This problem was partially solved by D.Mall(1991), J.J. Loeb and M. Nicolau(1996), T. Panov and Yu. Ustinovsky(2012).

There are quite many different complex structures on Calabi-Eckmann and Hopf manifolds, but not all of them can be obtained in the way that was proposed to construct complex structures on moment-angle manifolds in the paper of T. Panov and Yu. Ustinovsky. So there should be others complex structures. **The research problem** is to find some new complex structures on moment-angle manifolds and to study them.

In study of complex geometry it is interesting to describe an analytical subspaces of moment-angle manifolds. This problem was studied by T. Panov, Yu. Ustinovsky and M. Verbitsky (2013), L. Meersseman (2000). This problem was solved for some class of moment-angle manifolds by means of methods of Kähler geometry. **The problem** is to describe analytical subsets for a wider class of moment-angle manifolds.

The filtrability of coherent sheaf on Hopf manifold was proved in M. Verbitsky (2006). **The research problem** is to try to prove filtrability of coherent sheafs for some class of moment-angle manifolds.