SUMMARY

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The research is devoted to subexponential estimations in Shirshov's Height theorem. A word W is *n-divisible*, if it can be represented in the following form: $W = W_0 W_1 \cdots W_n$ such that $W_1 \prec W_2 \prec \cdots \prec W_n$, where $\prec$ is comparison in lexicographical sense. If an affine algebra A satisfies polynomial identity of degree n then A is spanned by non n-divisible words of generators $a_1 \prec \cdots \prec a_l$. A. I. Shirshov proved that the set of non n-divisible words over alphabet of cardinality l has bounded height h over the set Y consisting of all the words of degree $\leqslant n-1$.

We show, that $h < \Phi(n,l)$, where

$$\Phi(n,l) = 2^{96} l \cdot n^{12\log_3 n + 36\log_3\log_3 n + 91}.$$

Let l, n и $d \geqslant n$ be positive integers. Then all the words over alphabet of cardinality l which length is greater than $\Psi(n,d,l)$ are either n-divisible or contain d-th power of subword, where

$$\Psi(n,d,l) = 2^{27} l (nd)^{3\log_3(nd) + 9\log_3\log_3(nd) + 36}.$$

In 1993 E. I. Zelmanov asked the following question in Dniester Notebook:

"Suppose that $F_{2,m}$ is a 2-generated associative ring with the identity $x^m = 0$. Is it true, that the nilpotency degree of $F_{2,m}$ has exponential growth?"

We give the definitive answer to E. I. Zelmanov by this result. We show that the nilpotency degree of l-generated associative algebra with the identity $x^d = 0$ is smaller than $\Psi(d,d,l)$. This imply subexponential estimations on the nilpotency index of nil-algebras of an arbitrary characteristics. Original Shirshov's estimation was just recursive, in 1982 double exponent was obtained, an exponential estimation was obtained in 1992.

Our proof uses Latyshev idea of Dilworth's theorem application. We think that Shirshov's height theorem is deeply connected to problems of modern combinatorics. In particular this theorem is related to the Ramsey theory.

Above all we obtain lower and upper estimates of the number of periods of length $2, 3, (n-1)$ in some non n-divisible word. These estimates are differ only by a constant.