The purpose of the project is to study properties of groups of birational automorphisms of algebraic varieties on the level of their finite subgroups. Basically, I want to deal with two issues: general structure theorems about finite birational automorphism groups, and conjugacy classes of finite subgroups of groups of birational automorphisms.

In general, for any infinite group, one can wonder if its finite subgroups have some simple structure. The two inspiring cases are the groups $\mathrm{GL}_n(\Bbbk)$ for a finitely generated field $\Bbbk$ of characteristic 0 (say, $\Bbbk = \mathbb{Q}$), and $\mathrm{GL}_n(\Bbbk)$ for an arbitrary field $\Bbbk$ of characteristic 0 (say, $\Bbbk = \mathbb{C}$). It is known for a long time that any of the groups $\mathrm{GL}_n(\Bbbk)$ for a finitely generated field $\Bbbk$ of characteristic 0 has *bounded finite subgroups*, i. e. there is a constant $B = B(n, \Bbbk)$ such that any finite subgroup $G \subset \mathrm{GL}_n(\Bbbk)$ has order $|G| \leqslant B$. Also, by an old result of C. Jordan, the group $\mathrm{GL}_n(\mathbb{C})$ (and thus any group $\mathrm{GL}_n(\Bbbk)$ for a field $\Bbbk$ of characteristic 0) enjoys the so-called *Jordan property*, i. e. there is a constant $J_{\mathrm{GL}} = J_{\mathrm{GL}}(n)$ such that any finite subgroup $G \subset \mathrm{GL}_n(\mathbb{C})$ has a normal abelian subgroup $A \subset G$ of index $[G : A] \leqslant J_{\mathrm{GL}}$ (so that, informally speaking, all finite subgroups of $\mathrm{GL}_n(\mathbb{C})$ are essentially abelian).

Fix an arbitrary field $\Bbbk$ of characteristic 0. It was a recent expectation first expressed by J.-P. Serre that the above properties may hold for (many) groups of birational automorphisms of algebraic varieties. In particular, he showed that the group $\mathrm{Bir}(\mathbb{P}^2)$ of birational automorphisms of the projective plane is Jordan. Later Yu. Zarhin constructed an example of a surface X such that $\mathrm{Bir}(X)$ is not Jordan. V. Popov classified all surfaces with Jordan group of birational automorphisms.

Yu. Prokhorov and C. Shramov showed that modulo (conjectural) boundedness of terminal Fano n-folds the group of birational automorphisms of any n-dimensional rationally connected variety (in particular, the *Cremona group* $\mathrm{Cr}_n(\Bbbk) = \mathrm{Bir}(\mathbb{P}^n_{\Bbbk})$) is Jordan. Another thing that was done is an explicit (although absurdly large) estimate for the relevant constants in dimension 3. They Yu. Prokhorov and C. Shramov also made some advances in the case of arbitrary varieties.

Another thing that is of certain interest about finite groups of birational automorphisms is the structure of conjugacy classes. It is known that the question about conjugacy classes of subgroups of $\mathrm{Cr}_n(\Bbbk)$ isomorphic to G is naturally reinterpreted in terms of G-equivariant birational geometry of rational G-varieties. I. Dolgachev and V. Iskovskikh gave a classification of finite subgroups in the Cremona group $\mathrm{Cr}_2(\mathbb{C})$. Later Yu. Prokhorov found all finite simple non-abelian subgroups of $\mathrm{Cr}_3(\mathbb{C})$, proving the following. Actually, his results allow one (after some additional effort) to find all conjugacy classes of the “largest” finite simple non-abelian subgroups of $\mathrm{Cr}_3(\mathbb{C})$. Furthermore, I. Cheltsov and C. Shramov used the techniques of birational rigidity theory to produce a bunch of conjugacy classes of subgroups of Cr_3.

Within the current project I am going to study Jordan property for groups of birational automorphisms. In particular, I am going to try to find out how the geometry of degeneration locus of a conic bundle $X \to A$ influences the Jordan property of the group $\mathrm{Bir}(X)$. I am going to obtain an explicit estimate of a “multiplicative” analog of the Jordan constant for the group $\mathrm{Cr}_3(\mathbb{C})$ and apply it to study p-subgroups of $\mathrm{Cr}_3(\mathbb{C})$. I am also going to study Jordan property for automorphism groups of affine varieties and of compact complex surfaces. Finally, I am going to use techniques of birational rigidity theory together with cohomological obstructions to study conjugacy classes of subgroups of $\mathrm{Cr}_3(\mathbb{C})$, especially those that arise from G-Fano threefolds.