

**REPORT ON THE DYNASTY FELLOWSHIP FOR GRADUATE
STUDENTS AND YOUNG MATHEMATICIANS 2014**

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In 2014 I continued my study of extremal metrics for Laplace eigenvalues. For a fixed surface M one can consider eigenvalues λ_k as functionals on the space of Riemannian metrics of unit volume. By a famous result of Yang and Yau, for an orientable surface of genus γ one has an inequality

$$(1) \quad \lambda_1 \leq 8\pi \left[\frac{\gamma + 3}{2} \right].$$

Korevaar later generalized this result for an arbitrary functional λ_k , where 8π is replaced with a universal constant C independent of k and γ .

Extremal metrics for functionals λ_k are closely related to minimal surfaces in spheres. Basically any metric on a minimal surface M in the sphere is extremal for some functional λ_k , where k can be calculated in terms of the spectrum of M . In the paper [1] below we perform this calculation for a family of minimal surfaces in \mathbb{S}^4 .

Another project that was started in 2014 is proving an analog of inequality (1) for non-orientable surfaces. The paper is currently in preparation and is planned to be submitted during the winter term of 2015.

Papers

- [1] Spectral properties of bipolar surfaces to Otsuki tori.
Journal of Spectral Theory, 2014, Vol. 4, No. 1, pp. 87-111.

The i -th eigenvalue λ_i of the Laplace-Beltrami operator on a surface can be considered as a functional on the space of all Riemannian metrics of unit volume on this surface. Surprisingly only few examples of extremal metrics for these functionals are known. In the present paper a new countable family of extremal metrics on the torus is provided.

Scientific conferences and seminar talks

- [1] Conference "Lomonosov", Moscow, April 7 – 11.
Talk "Large Laplace and Steklov eigenvalues"
[2] Talk "Regularity theorems for maximal metrics" at "Spectral theory seminar" (McGill University, Montreal), November 27.