REPORT ON THE DINASTY–IUM FELLOWSHIP 2015

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Results

- We prove the conjecture by Feigin, Fuchs and Gelfand describing the Lie algebra cohomology of formal vector fields on an n-dimensional space with coefficients in symmetric powers of the coadjoint representation. We also compute the cohomology of the Lie algebra of formal vector fields that preserve a given flag at the origin. The latter encodes characteristic classes of flags of foliations and was used in the formulation of the local Riemann-Roch Theorem by Feigin and Tsygan.

  Feigin, Fuchs and Gelfand described the first symmetric power and to do this they had to make use of a fearsomely complicated computation in invariant theory. By the application of degeneration theorems of appropriate Hochschild-Serre spectral sequences we avoid the need to use the methods of FFG, and moreover we are able to describe all the symmetric powers at once.
- Given an operad $\mathcal{P}$ with a finite Groebner basis of relations, we study the generating functions for the dimensions of its graded components $\mathcal{P}(n)$. Under moderate assumptions on the relations we prove that the exponential generating function for the sequence $\dim \mathcal{P}(n)$ is differential algebraic, and in fact algebraic if $\mathcal{P}$ is a symmetrization of a non-symmetric operad. If, in addition, the growth of the dimensions of $\mathcal{P}(n)$ is bounded by an exponent of n (or a polynomial of n, in the non-symmetric case) then, moreover, the ordinary generating function for the above sequence $\dim \mathcal{P}(n)$ is rational. We give a number of examples of calculations and discuss conjectures about the above generating functions for more general classes of operads.
- The little cubes operad E_n governs natural operations one has on the iterated loop spaces. This operad is the most popular, however, we are far from understanding it in full details. In particular, it was proved recently by F.Brown that the set of homotopy infinitesimal automorphisms of this operad contains a free Lie algebra with infinite amount of generators and conjecturely (Deligne-Drinfeld conjecture) it is all. However, so far we do not know any reasonable conjecture about the description of cohomology classes of higher degrees in deformation complex.

  While inventing a new transgressive differentials we were able to prove the infinite-dimensionality of higher derivations of E_n and even compute some particular classes in small degrees. Moreover, we apply similar methods and prove infinite-dimensionality together with precise formulas of the cohomology of the deformation complex for the map between little cubes operads of different dimensions. The latter problem corresponds to the description of the rational homotopy type of the space of embedding $\mathbb{R}^m \hookrightarrow \mathbb{R}^n$ after Goodwillie–Weiss.

Papers

[1] "Characteristic classes of flags of foliations and Lie algebra cohomology."
Transformation Groups published online *December 1,2015*,

[2] with Dmitri Piontkovski
"On generating series of finitely presented operads."
Journal of Algebra 426 (2015) pp.377–429

[3] with T. Willwacher, M. Živković
"Differentials on graph complexes II – Hairy graphs"
preprint arxiv:1508.01281

Scientific conferences and seminar talks

[1] Conference "GRT, MZV and associators" August 20–29 in Les Diablerets (Switzerland)
Talk "Group actions, framed little balls operads and graph complexes"
[2] Visit Switzerland, February 2015,
Talks "On equivariant Deligne Conjecture" "Macdonald polynomials and Highest weight categories"
at *Séminaire Groupes de Lie et espace de modules*, Université de Génève;
Talk "Around categorification of Macdonald polynomial"
at *Talks in mathematical physics,* ETH Zurich;

Teaching

[1] Quantum Groups. Independent University of Moscow, III year and higher level students, September-December 2015, 4 hours per week (2 hours lecture + 2 hours seminar).

The goal of the coarse is to understand the notions and ideas invented by V.Drinfeld developed in the series of papers on quantum groups.

program:
- Coalgebras, Hopf algebras and tensor categories;
- Quantization and classical limit, Poisson algebras;
- Poisson-Lie groups, Lie bialgebras;
- Coboundary, triangular and quasitriangular Lie bialgebras;
- Drinfeld Double and universal R-matrix;
- Operad theory and Tamarkin's quantization;
- Little discs operad and Deligne's conjecture;
- Braided tensor categories;
- KZ-equations and Drinfeld category;