Summary of research

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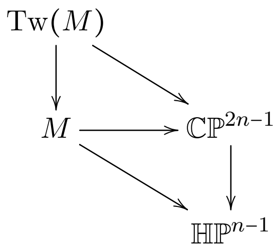
For my Master's thesis [2], I was given the task by Misha Verbitsky to generalize the result in [1] that twistor spaces of hyperkähler manifolds are balanced to general hypercomplex manifolds. This indeed proved to be correct, and the natural question to ask at this point would be whether the results of Kaledin and Verbitsky about the correspondence between moduli spaces of stable bundles on M and $\mathrm{Tw}(M)$ for a hyperkähler M generalize in some sense to hypercomplex manifolds.

One can start by investigating some particular examples of non-hyperkähler hypercomplex manifolds, such as the Hopf manifolds $M = (\mathbb{H}^n - \{0\})/\mathbb{Z}$. We have the diagram



where the vertical arrows are twistor projections and the horizontal arrow is a positive principal elliptic fibration, which, as shown in [3], establishes an intimate relationship between the moduli space of stable bundles of the manifolds M and $\mathbb{CP}^{2n-1}$. By studying the interplay between these moduli spaces, I would like to answer some questions such as whether a bundle on M which is stable for a given complex structure turns out to be stable for the other structures as well, or whether starting with a stable bundle on $\mathrm{Tw}(M)$, its restriction to a generic complex structure on M is stable.

References

[1] D. Kaledin, M. Verbitsky, "Non-Hermitian Yang-Mills connections", arXiv:alg-geom/9606019 (1996)

[2] A. Tomberg, "Twistor spaces of hypercomplex manifolds are balanced" (2014), preprint, arXiv:1409.1642, submitted to *Advances in Mathematics*

[3] M. Verbitsky, "Stable bundles on positive principal elliptic fibrations", arXiv:math/0403430 (2004)