Summary of the research statement of Mikhail V. Bondarko

Recently I proved: the weight complex functor of Gillet and Soule factors through Voevodsky motives. The new weight complex functor (from Voevodsky motives to complexes of Chow motives) converts "cones" of morphisms of motives into the cones of induced morphisms of their weight complexes, and does not send non-zero motives into contractible complexes.

It was proved that for any (co)homology of Voevodsky motives there exist functorial weight spectral sequences for any their cohomology (vastly generalizing Deligne's ones), whereas their E_2 -terms come from the "cohomology" of the corresponding weight complexes. I extended the statements mentioned to a general theory of weight structures for triangulated categories. The so-called Chow weight structures (for motives over fields and general base schemes) yield certain "weights" for motives (that "lift Deligne's weights"); the Gersten weight structures correspond to coniveau spectral sequences.

I would like to study various motivic categories, weight and t-structures for them further. In my last preprint I considered a certain "perverse" homotopy t-structure for motives over (a "reasonable") base scheme S. I would like to prove that the heart of this t-structure is the category of cycle modules over S (as defined by Rost). The interaction of this t-structure with the $Chow\ t$ -structure would give certain new results on unramified cohomology.

I will study effective Voevodsky motives in terms of certain Chow-weight homology obtained by applying the Chow functors (of cycles of a fixed dimension modulo rational equivalence) to the terms of weight complexes. For motives with rational coefficients over complex numbers (and other "large" algebraically closed fields) the zeroth Chow-weight homology groups vanish if and only if a motif is "divisible by the Lefschetz motif", i.e., if it vanishes "birationally". This gives a Chow-weight homology criterion for the divisibility of effective motives by a fixed power of the Lefschetz motif ("up to a given weight"). Dually, the Chow-weight cohomology measures a certain "dimension" of motives. Thus if the Chow-weight homology groups (with integral coefficients) are torsion in higher degrees, their exponent is necessarily bounded; they yield a criterion for the seminal higher motivic homology groups being torsion (which turns out to be equivalent to the boundedness of their exponents). Though Chow-weight (co)homology groups are rather difficult to calculate, they are somewhat easier to treat than the "ordinary" motivic (co)homology groups; they are known on the triangulated subcategory of motives generated by Tate twists of abelian varieties. One can also apply Chow-weight (co)homology for the study of motives (and cohomology) with integral coefficients; yet this requires considering Chow-weight homology over arbitrary function fields.

As a (somewhat) technical part of the project I will study weight and t-structures that can be described in terms of Chow motives. This would allow to relate motivic homology with the Chow-weight one, and yield a series of certain "mixed" weight structures.

Lastly, I would like to introduce a certain triangulated category of motives for complex analytic spaces that would contain the Voevodsky category of motives over the complex numbers, and introduce a weight structure for it that would extend the Chow weight structure.