SUMMARY

In the recent papers (see, e.g., [*Bourgain J., Korobkov M.V. and Kristensen J.*: *Rev. Mat. Iberoam. V.29, no. 1 (2013), 1–23*] and [*Journal fur die reine und angewandte Mathematik (Crelles Journal, Online first), DOI: 10.1515/crelle-2013-0002*]) we established Luzin N- and Morse–Sard properties for mappings $f\colon \mathbb{R}^n \to \mathbb{R}^m$ of the Sobolev–Lorentz class $W^k_{p,1}$ with $k=n-m+1$ and $p=\frac{n}{k}$ (this is the sharp case that guaranties the continuity of mappings). Using these results we prove that almost all level sets are finite disjoint unions of C^1–smooth compact manifolds of dimension $n-m$.

Using the two-dimensional version, i.e., for $W^2_1(\mathbb{R}^2)$, we proved the Bernoully Law under minimal smoothness assumptions (see [*Korobkov M.V.* Dokl. Math. 2011. V.83, No.1. P.107-110.]). We successfully applied this result to the following Leray's problem which remains open more then 80 years.

Consider the steady Navier–Stokes equations

$$\left\{\begin{aligned} -\nu\Delta\mathbf{u} + \big(\mathbf{u}\cdot\nabla\big)\mathbf{u} + \nabla p &= \mathbf{f} && \text{in } \Omega,\\ \operatorname{div}\mathbf{u} &= 0 && \text{in } \Omega,\\ \mathbf{u} &= \mathbf{a} && \text{on } \partial\Omega \end{aligned}\right. \tag{1}$$

in the domain $\Omega\subset\mathbb{R}^n$, $n=2,3$, with C^2-smooth boundary $\partial\Omega=\cup_{j=0}^N\Gamma_j$ consisting of $N+1$ disjoint components Γ_j, $j=0,\dots,N$. In (1) $\nu>0$ is the viscosity coefficient, $\mathbf{u}$, p are the (unknown) velocity and pressure fields, $\mathbf{a}$ is the (assigned) boundary data, and $\mathbf{f}$ is the body force density.

We proved the existence of the solutions to this problem for the following cases:

1) in bounded plane and axially symmetric spatial domains under necessary and sufficient conditions $\int_{\partial\Omega}\mathbf{a}\cdot\mathbf{n}\,dS=0$ (i.e., total flux is zero, see *Korobkov M.V., Pileckas K. and Russo R.* arXiv:1302.0731, to appear in **Annals of Math.**, http://annals.math.princeton.edu/articles/8861]);

2) in exterior axially symmetric spatial domains (see *Korobkov M.V., Pileckas K. and Russo R.* arXiv:1403.6921, submitted to *Acta Math.*]. (In the case of axially symmetric domains we assume that the boundary data, etc., are axially symmetric as well).

The main difference from the previous results is that no restrictions on the size of fluxes are assumed.

In our research we are going to concentrate on Leary's problems in bounded and exterior domains for the general spatial (3D) cases.