SUMMARY Zhukovskii Maksim Evgenievich

We study an asymptotical behaviour of first order properties of the Erdős–Rényi random graph $G(n,p)$. The random graph obeys the zero-one law if for each first-order property L either its probability tends to 0 or tends to 1. The random graph obeys the zero-one k-law if for each property L which can be expressed by first-order formula with quantifier depth at most k either its probability tends to 0 or tends to 1. Zero-one laws were proved for different classes of functions $p=p(n)$. The class $\{p=n^{-\alpha},\ \alpha>0\}$ is at the top of interest. In 1988 S. Shelah and J.H. Spencer proved that the random graph $G(n,n^{-\alpha})$ obeys zero-one law if $\alpha>0$ is irrational. If α is rational, $0<\alpha\leq 1$ then $G(n,n^{-\alpha})$ does not obey the zero-one law.

Let us denote the set of all first order properties and the set of all first order properties expressed by formulae with quantifier depth at most k by $\mathcal{L}$ and $\mathcal{L}_k$ respectively. For any $L\in\mathcal{L}$ we consider the set $S_1(L)$ of $\alpha\in(0,1)$ such that $\mathsf{P}_{n,n^{-\alpha}}(L)$ either does not converges or its limit is not zero or one. For any $L\in\mathcal{L}$ we also consider the set $S_2(L)$ of $\alpha\in(0,1)$ which does not satisfy the following property. There exist $\delta\in\{0,1\}$ and $\varepsilon>0$ such that for any $n^{-\alpha-\varepsilon}<p<n^{-\alpha+\varepsilon}$ equality $\lim_{n\to\infty}\mathsf{P}_{n,p}(L)=\delta$ holds. Set $S_1^k=\bigcup_{L\in\mathcal{L}_k}S_1(L)$, $S_2^k=\bigcup_{L\in\mathcal{L}_k}S_2(L)$.

Recently I proved that if $\alpha\in(0,\frac{1}{k-2})$ then the random graph $G(n,n^{-\alpha})$ obeys zero-one law. Moreover, $\frac{1}{k-2}\in S_k^1$. From this result it follows that the minimal number in S_1^k equals $\frac{1}{k-2}$. I also obtained the maximal number in S_1^k. Consider set $\mathcal{Q}$ of all positive rational numbers with the numerator less than or equal to 2^{k-1}. I also proved that the random graph $G(n,n^{-\alpha})$ obeys the zero-one k-law if $\alpha=1-\frac{1}{2^{k-1}+\beta}$, $\beta\in(0,\infty)\setminus\mathcal{Q}$. Moreover, $1-\frac{1}{2^{k-1}+\beta}\in S_1^k$ for any $\beta\in\{1,...,2^{k-1}-2\}$. If $\alpha\in\{1-\frac{1}{2^k},1-\frac{1}{2^k-1}\}$ then the random graph $G(n,n^{-\alpha})$ obeys the zero-one k-law as well. So, the maximal number in S_k^1 equals $1-\frac{1}{2^k-2}$.

J.H. Spencer in 1990 proved that sets S_k^1 and S_k^2 are infinite when k is large enough. Moreover, in 1991 he proved that all limit points of S_k^1 and S_k^2 are approached only from above (in other words, the set S_2^k is well-ordered under $>$). At the present time in our joint work with J.H. Spencer we receive some new results: find the maximal and the minimal numbers in S_k^2; estimate the maximal and the minimal limit points in S_k^1 and S_k^2; for each $j\in\{1,2\}$ estimate the minimal k such that S_k^j is infinite.

In future, for each $j\in\{1,2\}$ I am going to find the the exact values of the maximal and the minimal limit points in S_k^j and find the exact value of the minimal k such that S_k^j is infinite. Moreover, I want to find the number of values in S_k^1, S_k^2 near there minimal and maximal values. In other words, if the sets $S_k^j\cap\left(0,\frac{1}{k-2.5}\right)$, $S_k^j\cap\left(1-\frac{1}{2^{k-1}},1\right)$, $j\in\{1,2\}$, are finite, I'm going to find its power. I want to prove zero-one k-laws for $G(n,n^{-\alpha})$ for new sets of rational α (especially, for small k). Finally, I am going to investigate zero-one laws and zero-one k-laws for random uniform hypergraphs (all edges of the complete hypergraph appear in the random graph with probability p mutually independently).