$$\sin^2 x + \cos^2 x = 1 \tag{1}$$

$$\sin x + \cos x = 1 \tag{2}$$

$$\tan^2 x + \cot^2 x = 1 \tag{3}$$

$$\forall x \ \ \sin^2 x + \cos^2 x = 1 \tag{4}$$

$$\exists x \ \ \sin x + \cos x = 1 \tag{5}$$

$$\neg\exists x \ \ \tan^2 x + \cot^2 x = 1 \tag{6}$$

$$\mathbf{+ \quad {}^2 \quad sin \quad cos \quad tan \quad cot} \tag{7}$$

$$3 = 7 \tag{8}$$

$$x = 7, \ \ x + y = 7, \ \ \exists y(x + y = 7) \tag{9}$$

$$\mathbf{E}: \qquad \forall x \forall y \, x = y \tag{10}$$

$$\mathrm{Sp}(\mathbf{F}) \tag{11}$$

$$\exists\varphi\ \Big[\forall x_1\forall x_2\big(\varphi(x_1)=\varphi(x_2)\Rightarrow x_1=x_2\big)\ \wedge \\ \wedge\ \exists y\forall x\ \varphi(x)\neq y\Big] \tag{12}$$

$$\exists w\exists\psi\ \Big(\forall x[w\neq\psi(x)]\ \wedge \\ \wedge\ \forall x_1\forall x_2\big[\overbrace{\psi(x_1)=\psi(x_2)\Rightarrow x_1=x_2}\big]\Big) \tag{13}$$

$$o,\quad \psi(o),\quad \psi(\psi(o)),\quad \psi(\psi(\psi(o))),\quad \ldots \tag{14}$$

$$\mathbf{A}_2:\quad \exists x_1\exists x_2\, x_1\neq x_2 \tag{15}$$

$$\mathrm{Sp}(\mathbf{A}_2)\ =\ \{\varkappa\mid\varkappa\geqslant 2\} \tag{16}$$

$$\mathbf{A}_n:\quad \exists x_1\exists x_2\ \ldots\ \exists x_n\big(\\ x_1\neq x_2\ \wedge\ x_1\neq x_3\ \wedge\ \ldots\ \wedge\ x_1\neq x_n\ \wedge \\ \wedge\, x_2\neq x_3\ \wedge\ \ldots\ \wedge\ x_2\neq x_n\ \wedge\ \ldots\ \wedge\ x_{n-1}\neq x_n\big) \tag{17}$$

$$\mathrm{Sp}(\ \mathbf{A}_n)\ =\ \{\varkappa\mid\varkappa\geqslant n\} \tag{18}$$

$$\exists x_1\ \ldots\ \exists x_n\ \bigwedge_{1\leq i<j\leq n} x_i\neq x_j \tag{19}$$

$$\mathbf{B}_n : \qquad \mathbf{A}_n \wedge \neg\mathbf{A}_{n+1} \tag{20}$$

$$\mathrm{Sp}(\mathbf{B}_n) = \{n\} \tag{21}$$

$$\{a,\ b,\ c,\ \dots,\ l\} \tag{22}$$

$$\mathbf{B}_{\{a,\ b,\ c,\ \dots,\ l\}} =$$

$$\mathbf{B}_a \vee \mathbf{B}_b \vee \mathbf{B}_c \vee \dots \vee B_l \tag{23}$$

$$\mathrm{Sp}(\mathbf{B}_{\{a,\ b,\ c,\ \dots,\ l\}}) = \{a,\ b,\ c,\ \dots,\ l\} \tag{24}$$

$$\neg\mathbf{B}_{\{a,\ b,\ c,\ \dots,\ l\}} = \mathbb{N} \setminus \{a,\ b,\ c,\ \dots,\ l\} \tag{25}$$

$$\begin{aligned} &\exists w \exists \psi\ \forall x [w \neq \psi(x)] \wedge \\ &\wedge \forall x_1 \forall x_2 [\psi(x_1) = \psi(x_2) \Rightarrow x_1 = x_2] \wedge \\ &\wedge \forall y \exists x [y = \psi(x)] \end{aligned} \tag{26 ${}_{13}$}$$

$$\begin{aligned} &\exists w \exists \psi\ \forall x (w \neq \psi(x)) \wedge \\ &\wedge \forall x_1 \forall x_2 (\psi(x_1) = \psi(x_2) \Rightarrow x_1 = x_2) \wedge \\ &\wedge \forall W \Big(\overbrace{W(w) \wedge \forall x [\overbrace{W(x) \Rightarrow W(\psi(x))}] \Rightarrow \forall x W(x)} \Big) \end{aligned} \tag{27${}_{13}$}$$

Аксиомы линейного порядка

H1. $\forall x \forall y \forall z (x \prec y \wedge y \prec z \Rightarrow x \prec z)$.

H2. $\forall x \neg (x \prec x)$.

H3. $\forall x \forall y (x \prec y \vee y \prec x \vee x = y)$.

Дедекиндово сечение

H4. $\forall P \Big[\Big(\overbrace{\exists x P(x) \wedge \exists y \neg P(y) \wedge \forall x \forall y \big[(P(x) \wedge \neg P(y)) \Rightarrow x \prec y \big]} \Big) \Rightarrow$

$\Rightarrow \Big(\Big[\exists u \big(\overbrace{P(u) \wedge \forall x [\overbrace{P(x) \Rightarrow (x \prec u \vee x = u)}]} \big) \vee$

$\vee \exists v \big(\overbrace{\neg P(v) \wedge \forall y [\overbrace{\neg P(y) \Rightarrow (v \prec y \vee v = y)}]} \big) \Big] \wedge$

$\wedge \neg \Big[\exists u \big(\overbrace{P(u) \wedge \forall x [\overbrace{P(x) \Rightarrow (x \prec u \vee x = u)}]} \big) \wedge$

$\wedge \exists v \big(\overbrace{\neg P(v) \wedge \forall y [\overbrace{\neg P(y) \Rightarrow (v \prec y \vee v = y)}]} \big) \Big] \Big) \Big]$

Объём свойства Q плотен в носителе, или, неформально, множество Q плотно в носителе

H5. $\forall x \forall z \big[x \prec z \Rightarrow \exists y \big(Q(y) \wedge x \prec y \wedge y \prec z \big) \big]$.

Для множества Q как бы выполняются аксиомы натурального ряда

H6. $Q(0_Q)$.

H7. $\forall x(Q(x) \Rightarrow Q(x'))$.

H8. $\neg\exists x(Q(x) \wedge x' = 0_Q)$.

H9. $\forall x \forall y \Big[\big(Q(x) \wedge Q(y) \wedge (x' = y')\big) \Rightarrow (x = y)\Big]$.

$$\text{H10. } \forall P\Bigg(\Big[P(0_Q) \wedge \forall x\big([Q(x) \wedge P(x)] \Rightarrow P(x')\big)\Big] \Rightarrow$$
$$\Rightarrow \forall x[Q(x) \Rightarrow P(x)]\Bigg).$$

Любое надмножество множества Q является равномощным либо Q, либо всему носителю

$$\text{H11. } \forall W\Bigg(\Big[\overbrace{\forall x\big(Q(x) \Rightarrow W(x)\big)}\Big] \Rightarrow$$
$$\Rightarrow \exists\varphi\Bigg[\Bigg(\forall y \exists x\Big[\overbrace{W(x) \wedge (\varphi(x) = y)}\Big] \wedge$$
$$\wedge \forall y \forall x_1 \forall x_2\Big[\Big(\overbrace{W(x_1) \wedge W(x_2) \wedge (\varphi(x_1) = y) \wedge (\varphi(x_2) = y)}\Big) \Rightarrow$$
$$\Rightarrow (x_1 = x_2)\Big]\Bigg) \vee$$
$$\vee \Bigg(\forall x\Big[\overbrace{W(x) \Rightarrow Q(\varphi(x))}\Big] \wedge$$
$$\wedge \forall y\Big[\overbrace{Q(y) \Rightarrow \exists x\big(\overbrace{W(x) \wedge (\varphi(x) = y)}\big)}\Big] \wedge$$
$$\wedge \forall y \forall x_1 \forall x_2\Big[\Big(\overbrace{W(x_1) \wedge W(x_2) \wedge (\varphi(x_1) = y) \wedge (\varphi(x_2) = y)}\Big) \Rightarrow$$
$$\Rightarrow (x_1 = x_2)\Big]\Bigg)\Bigg]\Bigg).$$