George Shabat 70 Conference

Moscow, MCCME, 21-22.06.2022

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1 Nikolai Adrianov
Enumerating dessins with almost regular elliptic passports

We consider passports that are close to regular elliptic passports of types (3,3,3), (2,4,4) and (2,3,6). Running computer-based enumeration of dessins with these passports we guess a number of nice sequences with generating functions related to theta-functions. Some of these experimental results will be proved. Many other formulas and observations will be stated as conjectures.

2 Natalia Amburg
Tensor models and ramified covering of 3D sphere

We consider rank 3 random tensor model with Gaussian measure. Integral of an operator by Wick theorem defines ramified covering of 3D sphere. 3D covering space is glued from tetrahedrons.

3 Boris Bychkov, Petr Dunin-Barkowski
Double Hurwitz numbers, maps, topological recursion, and dualities

There are many enumerative problems whose answers are encoded in the Taylor coefficients of a sequence of the so-called m-point correlator functions. The topological recursion (due to Chekhov-Eynard-Orantin) is an inductive procedure for explicit computation of these functions in a closed form starting from a relatively small amount of initial data. An extension of this problem leads to a collection of (m,n)-point correlator functions such that the original ones correspond to the case n=0. It proves that the sequence of (0,n) functions also satisfies its own topological recursion with its own initial data. This fact was known before for the two-matrix model related to the problem of enumeration of (hyper-) maps. The two recursions are related in this case by the x-y duality which is well studied in a general formalism of topological recursion. We generalize this to the case of generating functions of generalized double weighted Hurwitz numbers where points over zero and infinity can be both marked and unmarked. We find an explicit recursion formula for the (m,n)-functions and prove the respective generalizations of the loop equations and the projection property for them. This is a work in progress by B. Bychkov, P. Dunin-Barkowski, M. Kazarian, and S. Shadrin.

4 Leonid Chekhov
Cluster algebras and integrability

Cluster algebras provide an amazingly simple and explicit description of canonical coordinates/mapping class group transformations for spaces of SL_n character varieties on Riemann surfaces with holes. I will show how we can use the system on a disc with three marked points on its boundary to describe solutions for the quantum reflection equation whose particular Poisson leaves are related to geodesic functions on Riemann surfaces with holes. Based on a joint paper with M. Shapiro.

5 Anna Felikson
Friezes for a pair of pants

Frieze patterns are numerical arrangements that satisfy a local arithmetic rule. Conway and Coxeter showed that frieze patterns are tightly connected to triangulated polygons. Recently, friezes were actively studied in connection to the theory of cluster algebras, and the notion of a frieze obtained a number of generalisations. In particular, one can define a frieze associated with a bordered marked surface endowed with a decorated hyperbolic metric. We will review the construction and will show that some nice properties can be extended to friezes associated to a pair of pants. This work is joint with Ilke Canakci, Ana Garcia Elsener and Pavel Tumarkin, arXiv:2111.13135.

6 Vladimir Gurvich
On the degree sequences of dual graphs on surfaces

Endre Boros, Vladimir Gurvich, Martin Milanič, Jernej Vičič

Given two graphs G and $G*$ with a one-to-one correspondence between their edges, when do G and $G*$ form a pair of dual graphs realizing the vertices and countries of a map embedded in a surface? A criterion was obtained by Jack Edmonds in 1965. Furthermore, let $d = (d_1, \ldots, d_n)$ and $t = (t_1, \ldots, t_m)$ be their degree sequences. Then, clearly, $\sum_{i=1}^{n} d_i = \sum_{j=1}^{m} t_j = 2\ell$, where ℓ is the number of edges in each of the two graphs, and $\chi = n - \ell + m$ is the Euler characteristic of the surface.

Which sequences d and t satisfying these conditions still cannot be realized as the degree sequences?

We make use of Edmonds' criterion to obtain several infinite series of exceptions for the sphere, $\chi = 2$, and projective plane, $\chi = 1$. We conjecture that there exist no exceptions for $\chi \leq 0$ and can prove this if every degree is at least 10. Note however that for $\chi = 0$ exceptions exist for torus and for Klein bottle, but not for both simultaneously.

7 Alexander Guterman
Maps preserving matrix invariants

The first results on transformations preserving matrix invariants is due to Frobenius. This result describes the structure of linear maps T preserving the determinant function, i.e., $\det X = \det T(X)$ for all X. Later on there were several extension of this result which are due to Diedonnie, Schur, Dynkin and others. Later on different maps preserving matrix properties, invariants, relations on operator or matrix algebras over various algebraic structures were investigated by many researchers . We plan to discuss the corresponding problems for matrices over division rings and characterize semilinear maps preserving Dieudonné determinant and singularity. This provides non-commutative analogs of Frobenius and Dieudonné theorems.

8 Yulij Ilyashenko
Simultaneous uniformization of the leaves of analytic foliations

The main problem discussed in this talk is: *Given an analytic family of Riemann surfaces, find a uniformization of these surfaces analytically depending on the parameter of the family.* The term *Simultaneous uniformization* is due to Bers, as well as the first positive results in the study of the problem above. The talk will contain a survey of the positive and negative results on the problem stated above due to various authors, as well as some new results due to A. Shcherbakov, to the speaker and to the hero of this conference. Open problems will be stated.

9 Gareth Jones
Operations on maps and dessins

For a given finite group G the orientably regular maps (or, more generally, the regular dessins) with automorphism group G are used as the vertices of a graph $\mathcal{O}(G)$, with undirected and directed edges showing the effect on these maps of various operations, such as duality and Coxeter's hole operations. Examples of these graphs are given, including several for small Hurwitz groups. The connected components of $\mathcal{O}(G)$ consist of maps (or dessins) which can be mutually transformed into each other by these operations. For some G, such as the affine groups $\mathrm{AGL}_1(2^e)$, the graph is connected, whereas for the alternating and symmetric groups the number of connected components is shown to be unbounded. This is joint work with Gábor Gévay, based on a paper to appear in *ADAM*.

10 Anton Khoroshkin
On Kontsevich Graph complexes

The Kontsevich graph complex GC_2 has a simple combinatorial description and very complicated cohomology. Its 0'th cohomology coincides with the completion of the Grothendieck-Teichmuller Lie algebra grt and the first cohomology conjecturally vanishes.

I recall the definition of Kontsevich Graph complexes, explain infinite-dimensionality of cohomology and give different related combinatorial models, that are necessary for applications for universal quantization theorem.

11 Elena Kreines
On simple assembly graphs

Doubly occurrence words play important role in genetics. They can be described in terms of so-called simple assembly graphs. We describe and investigate several properties of these graphs and related dessins d'enfants.

12 Alexander Molyakov
Endomorphisms of the loop group of multiplicative group

The classical Contou-Carrère symbol is a remarkable bilinear pairing defined on the group of invertible elements of $A((t))$, the ring of formal Laurent series with coefficients in a ring A. The pairing is skew-symmetric and satisfies the Steinberg property. When the base ring A is a field it coincides with the usual tame symbol. The Contou-Carrère symbol has a certain universal property which is implied by Contou-Carrère theorem. I will discuss a recently obtained analogue of this theorem for the loop group of the multiplicative group over a $\mathbb{Q}$-algebra. The result allows one to describe endomorphisms of this loop group in terms of double Laurent series of a special form.

13 Daniil Rudenko

Volumes of hyperbolic polytopes, cluster polylogarithms, and the Goncharov depth conjecture

Lobachevsky started to work on computing volumes of hyperbolic polytopes long before the first model of the hyperbolic space was found. He discovered an extraordinary formula for the volume of an orthoscheme via a special function called dilogarithm. We will discuss a generalization of the formula of Lobachevsky to higher dimensions. For reasons I do not fully understand, a mild modification of this formula leads to the proof of a conjecture of Goncharov about the depth of multiple polylogarithms.

14 George Shabat
On generalizations of dessins d'enfants and Belyi pairs

I understand dessins d'enfants theory as the three-language dictionary (graphs on surfaces, curves over number fields, 2-generated groups). In the talk I am going to omit groups and discuss the generalizations of dessins $X_0 \subset X_1 \subset X_2$ and Belyi pairs $(\mathbf{X}, \beta)$. A generalization will be considered successful if some interesting mathematics emerges.

We shall briefly discuss such generalizations as (1) singular X_2's, (2)allowing $X_2 \setminus X_1$ to contain annuli components, (3) Fried families, (4) Belyi pairs over various fields (5) transcendent Belyi functions.

The main attention will be paid to the multi-dimensional generalizations, starting with $X_0 \subset X_1 \subset \cdots \subset X_n$. For $n = 3$ a Yu.I. Manin's idea of considering the geometry inside the handlebodies will be reminded together with its relations to the Arakelov geometry. For $n = 4$, i.e. $\dim_{\mathbb{C}} X_4 = 2$, some analogs of the Belyi pairs for algebraic surfaces will be discussed.

15 Sergei Smirnov
Integral preserving discretization of two-dimensional Toda lattices

Toda lattices play an important role both in differential geometry and in mathematical physics. Exponential systems that generalize two-dimensional Toda lattice were studied in a number of papers by A. B. Shabat, A. N. Leznov, A. V. Mikhailov and by many others. Exponential systems corresponding to the Cartan matrices of simple Lie algebras are known to be Darboux integrable. Various discrete versions of Toda lattices were introduced during the last two decades. Systematic approach to discretizing exponential systems was proposed by Habibullin in 2011: the idea is to look for (semi)-discrete systems of a certain type having the same characteristic integrals, as the continuous model.

We prove that if an exponential system is Darboux integrable in the continuous case, then its Habibullin's integral preserving discretization is also Darboux integrable in the semi-discrete case. In particular, this allows to prove Darboux integrability of semi-discrete Toda lattices corresponding to the Cartan matrices of the series B and D.

16 Alexander Tumanov
Finite jet determination for CR mappings

A CR mapping is a diffeomorphism between two real manifolds in complex space that satisfies tangential Cauchy-Riemann equations. We are concerned with the problem whether a CR mapping is uniquely determined by its finite jet at a point. This problem has been popular since 1970-s and the number of publications on the matter is enormous. Nevertheless, natural fundamental questions have been open. I will present a solution to a version of the problem and discuss old and recent results.

17 Pavel Tumarkin
Quivers, surfaces, and reflection groups

Given an unpunctured triangulated marked surface with boundary, we can associate with it two groups: an extended affine Weyl group of type A and a certain group which behaves nicely with respect to changes of triangulations. I will discuss a connection between these groups and some applications of the constructions. The talk is based on joint works with Anna Felikson, John Lawson and Michael Shapiro.

18 Alexander Voronov
On the BV structure on the cohomology of moduli space

George has always been fascinated with the moduli space of algebraic curves and inspired others to look at it from an unusual angle. The cohomology of the moduli space, apart from the standard graded commutative algebra structure, acquires a BV-algebra structure, when you take into account curves of various genera and with arbitrarily many punctures. We will describe this BV structure, which comprises a BV operator and antibracket, prove vanishing theorems for it, and provide a counterexample to a physically motivated conjecture. This is a joint work with S. Sakalli, On the BV structure on the cohomology of moduli space, Pure Appl. Math. Q. 16 (2020), no. 3, 755–773.

19 Dimitri Zvonkine
How quantum Hall effect leads to algebraic geometry

The Hall effect is observed when a thin conducting plate is traversed by a strong magnetic field. Moving charged particles are deviated by the field, so that a difference of potentials along the x-axis causes an electric current not only in the x-direction, but also in the y-direction. As true mathematicians we study the Hall effect on a Riemann surface. This leads to investigating vector bundles over the Picard group of the surface. One of the physically meaningful problems is to find the Chern classes of these vector bundles. Since this is a short talk, I will not present the mathematical results, but instead will explain how a concrete physical problem leads to serious algebraic geometry. This is joint work with Semyon Klevtsov.

20 Conference Program

June 21, 2022

Chairman	George Shabat	
12:00 - 12:30	George Shabat	in person
12:30 - 13:00	Nikolai Adrianov	in person
13:00 - 13:30	Gareth Jones	zoom
13:30 - 14:00	Coffee break	
Chairman	Gareth Jones	
14:00 - 14:30	Elena Kreines	zoom
14:30 - 15:00	Vladimir Gurvich	zoom
15:00 - 15:30	Natalia Amburg	in person
15:30 - 16:30	Break	
Chairman	Alexander Guterman	
16:30 - 17:00	Boris Bychkov	zoom
17:00 - 17:30	Petr Dunin-Barkowski	in person
17:30 - 18:00	Anton Khoroshkin	zoom
18:00 - 18:30	Alexander Voronov	zoom
18:30 - 19:00	General Discussion	
19:00 - ...	Conference Dinner	

June 22, 2022

Chairman	Nikolai Adrianov	
11:00 - 11:30	Leonid Chekhov	zoom
11:30 - 12:00	Yulij Ilyashenko	zoom
12:00 - 12:30	Alexander Molyakov	in person
12:30 - 13:00	Coffee break	
Chairman	Elena Kreines	
13:00 - 13:30	Anna Felikson	zoom
13:30 - 14:00	Pavel Tumarkin	zoom
14:00 - 14:30	Alexander Guterman	zoom
14:30 - 15:30	Break	
Chairman	George Shabat	
15:30 - 16:00	Sergei Smirnov	in person
16:00 - 16:30	Alexander Tumanov	zoom
16:30 - 17:00	Dimitri Zvonkine	zoom
17:00 - 17:30	Daniil Rudenko	zoom
17:30 - 18:30	Concluding Remarks	

You are welcome, ZOOM: 837 2639 6706; password: 6 digits of π.