# The IUM report to the Simons foundation, 2018

## Contents

# 1 Introduction: list of awardees

The Simons foundation supported two programs launched by the IUM:
Simons stipends for students and graduate students;
Simons IUM fellowships.

12 applications were received for the Simons stipends contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, G.Kabatyanski, S.Lando, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman* awarded Simons stipends for 2018 year to the following students and graduate students:

1. Abramyan Semyon Arturovich
2. Bogachev, Nikolay Vladimirovich
3. Ilyin, Alexei Igorevich
4. Ivanov, Alexei Nikolaevich
5. Kalmynin, Alexander Borisovich
6. Kirillov, Ilya Victorovich
7. Koshelev, Dmitry Igorevich
8. Loginov, Konstantin Valerevich
9. Osipov,Pavel Sergeevich

13 applications were received for the Simons IUM fellowships contest for the first half year of 2018 and 14 applications were received for the second half year. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, B.Feigin, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman, V.Vassiliev* awarded

Simons IUM-fellowships for the first half year of 2018 to the following researches:

1. Belavin, Alexander Abramovich
2. Elagin, Alexei Dmitrievich
3. Fedorovsky, Konstantin Yuryevich
4. Fonarev, Anton Vyacheslavovich
5. Panov, Taras Evgenyevich

6. Penskoi, Alexei Victorovich
7. Rybnikov, Leonid Grigoryevich
8. Shabat, George Borisovich
9. Shaposhnikov, Stanislav Valeryevich
10. Smirnov, Evgeni Yurevich
11. Vyugin, Ilya Vladimirovich
12. Zhgoon, Vladimir Sergeevich

Simons IUM-fellowships for the second half year of 2018 to the following researches:

1. Belavin, Alexander Abramovich
2. Burman, Yurii Mikhailovich
3. Elagin, Alexei Dmitrievich
4. Kolesnikov, Alexander Victorovich
5. Lashkevich, Mikhail Yuryevich
6. Panov, Taras Evgenyevich
7. Penskoi, Alexei Victorovich
8. Poberezhnyi, Vladimir Andreevich
9. Rybnikov, Leonid Grigoryevich
10. Shabat, George Borisovich
11. Shaposhnikov, Stanislav Valeryevich
12. Skopenkov, Mikhail Borisovich
13. Smirnov, Evgeni Yurevich

The report below is split in two sections corresponding to the two programs above. The first subsection in each section is a report on the research activities. It consists of the titles of the papers published or submitted in the year of 2018, together with the corresponding abstracts. The second subsection of each section is devoted to conference and some most important seminar talks. The last subsection of the second section is devoted to the syllabi of the courses given by the winners of the Simons IUM fellowships. Most of these courses are innovative, as required by the rules of the contest for the Simons IUM fellowships.

The Independent University remains one of the most active centers of Moscow Mathematical life. There is no room here to list its main activities. We only mention that the two invited sectional speakers of the ICM-2018 from Moscow, M. Finkelberg and A. Belavin, are permanent Professors of the IUM. Moreover, the plenary speaker from Russia is Andrei Okounkov with a double affiliation: Russia and USA. His Russian affiliation is related to the IUM in the following way: the IUM created, together with the Higher School of Economy, a new Department of Mathematics, and this department created an International Laboratory of Representation Theory and Mathematical Physics, with Andrei Okounkov as a director.

The support of the Simons foundation have drastically improved the financial situation at the IUM, and the whole atmosphere as well. On behalf of the IUM, I send my deep gratitude and the best New year wishes to Jim Simons, Yuri Tschinkel, and the whole team of the Simons foundation.

Yulij Ilyashenko

President of the Independent University of Moscow

# 2 Program: Simons stipends for students and graduate students

## 2.1 Research

### 2.1.1 Semyon Abramyan

[1] Iterated Higher Whitehead products in topology of moment-angle complexes arXiv:1708.01694 *accepted by Siberian Mathematical Journal*

In this paper we study the topological structure of moment-angle complexes $\mathcal{Z}_{\mathcal{K}}$. We consider two classes of simplicial complexes. The first class $B_\Delta$ consists of simplicial complexes $\mathcal{K}$ for which $\mathcal{Z}_{\mathcal{K}}$ is homotopy equivalent to a wedge spheres. The second class $W_{Delta}$ consists of $\mathcal{K} \in B_\Delta$ such that all spheres in the wedge are realised by iterated higher Whitehead products. Buchstaber and Panov asked if it is true that $W_\Delta = B_\Delta$. In this paper we show that this is not the case. Namely, we give an example of a simplicial complex whose corresponding moment-angle complex is homotopy equivalent to a wedge of spheres, but there is a sphere which cannot be realised by any linear combination of iterated higher Whitehead products. On the other hand we show that class $W_\Delta$ is large enough. Namely, we show that the class $W_\Delta$ is closed with respect to two explicitly defined operations on simplicial complexes. Then using these operations we prove that there exists a simplicial complex that realises any given iterated higher Whitehead product. Also we describe the smallest simplicial complex that realises an iterated product with only two pairs of nested brackets.

[2] With T. Panov
Whitehead products in moment-angle complexes and substitution of simplicial complexes (in preparation)

We describe the simplicial complex $\partial\Delta_w$ which realises a given general iterated higher Whitehead bracket $w \in \pi(\mathcal{Z}_{\mathcal{K}})$. For a particular form of brackets inside $w$, we prove that $\partial\Delta_w$ is the smallest complex that realises $w$. We describe the canonical cellular chain which represents the Hurewicz image of a general iterated higher Whitehead product $w$. Using coalgebraic Taylor resolution we describe another canonical representative for the Hurewicz image of a general iterated higher Whitehead product $w$ in terms of missing faces of simplicial complex $\mathcal{K}$.

### 2.1.2 Nikolay Bogachev

[1] With A. Perepechko
Vinberg's Algorithm for Hyperbolic Lattices
*Mathematical Notes, 2018, Vol. 103:5, pp. 836–840*

A hyperbolic lattice (that is a lattice of signature $(n, 1)$) is said to be reflective if the subgroup of its automorphism group generated by all reflections is of finite index. It is well known that reflective hyperbolic lattices exist only for $n < 22$ and there are only finitely many of them. But the classification problem is now completely solved only for $n = 2$ and $n = 4, 5$.

In 1972 Vinberg proposed an algorithm that, given a lattice, enables one to find recursively all faces of the fundamental polyhedron of the corresponding reflection subgroup, determine if there are only finitely many of them and hence, to verify this lattice on reflectivity.

We present in this paper the software implementation of this algorithm, first implemented for hyperbolic lattices of arbitrary form, as well as new reflective hyperbolic lattices, which were found by this program. The program was tested by the large number of hyperbolic lattices.

[2] Classification of (1,2)-reflective anisotropic hyperbolic lattices of rank 4
*to appear in Izvestiya Mathematics, 2019, Vol. 83:1, pp. 3–24.*

A hyperbolic lattice is called (1,2)-*reflective* if the subgroup of its automorphism group generated by all 1- and 2-reflections is of finite index. In this paper we prove that the fundamental polyhedron of any $\mathbb{Q}$-arithmetic cocompact reflection group in the three-dimensional hyperbolic space contains an edge such that the distance between the framing faces of this edge is small enough. Using this fact we obtain a complete classification of (1,2)-reflective anisotropic hyperbolic lattices of rank 4.

[3] Reflective hyperbolic lattices
*PhD thesis, HSE, 2019, February*

This thesis is devoted to classification of reflective hyperbolic lattices, which is an open problem since the 1970s. The dissertation was written under the supervision of Professor E. B. Vinberg during my postgraduate study at the Department of Higher Algebra of the Faculty of Mechanics and Mathematics of the Moscow State University.

The thesis consists of five chapters. The first chapter is an introduction to the subject. It includes only some basic definitions along with a number of known facts and open problems as well as the formulations of the main results of the dissertation. Chapter 2 contains some auxiliary results, including a description of models of spaces of constant curvature, acute-angled polyhedra in them, discrete groups of reflections, and, finally, the fundamentals of the theory of reflective hyperbolic lattices and arithmetic hyperbolic reflection groups.

The main results of this dissertation are obtained in Chapters 3, 4, and 5. Chapter 3 gives a theoretical description of Vinberg's Algorithm and also the description of the project (our joint work with A. Yu. Perepechko) VinAl of a computer implementation of Vinberg's Algorithm for hyperbolic lattices over $\mathbb{Z}$. Finally, Chapters 4 and 5 contain the results of classification of stably reflective hyperbolic lattices of rank 4 over $\mathbb{Z}$ and $\mathbb{Z}[\sqrt{2}]$, respectively.

[4] Classification of stably reflective hyperbolic lattices of rank 4 over $\mathbb{Z}[\sqrt{2}]$.
*in progress*

In this paper we prove that the fundamental polyhedron of any $\mathbb{Q}[\sqrt{2}]$-arithmetic cocompact reflection group in the three-dimensional hyperbolic space contains an edge such that the distance between the framing faces of this edge is small enough. Using this fact we obtain a complete classification of stably reflective hyperbolic lattices of rank 4 over $\mathbb{Z}[\sqrt{2}]$.

[5] Bounds for degrees of ground fields of arithmetic hyperbolic reflection groups
*in progress*

In this paper we prove that any compact hyperbolic Coxeter polyhedron contains an edge such that the distance between the framing faces of this edge is small enough. Using this fact we improve the upper bounds for degrees of ground fields of arithmetic hyperbolic reflection groups.

#### 2.1.3 Alexei Ilyin

[1] Aleksei Ilin, Leonid Rybnikov *Bethe subalgebras in Yangians and the wonderful compactification.*
arXiv:1810.07308, submitted to Communications in Mathematical Physics.

In this paper we study the family of Bethe subalgebras in the Yangian $Y(\mathfrak{g})$ parameterized by the corresponding adjoint Lie group $G$. We describe their classical limits as subalgebras in the algebra of polynomial functions on the formal Lie group $G_1[[t^{-1}]]$. In particular we show that, for regular values of the parameter, these subalgebras are free polynomial algebras with the same Poincare series as the Cartan subalgebra of the Yangian. Next, we extend the family of Bethe subalgebras to the De Concini–Procesi wonderful compactification $\overline{G} \supset G$ and describe the subalgebras corresponding to generic points of any stratum in $\overline{G}$ as Bethe subalgebras in the Yangian of the corresponding Levi subalgebra in $\mathfrak{g}$. In particular, we describe explicitly all Behte subalgebras corresponding to the closure of the maximal torus in the wonderful compactification.

#### 2.1.4 Alexei Ivanov

[1] With A. S. Tikhomirov
Semistable rank 2 sheaves with singularities of mixed dimension on $\mathbb{P}^3$
J. Geometry and Physics, 2018, Vol. 129, pp. 90-98.

In this paper, we describe new irreducible components of the Gieseker-Maruyama moduli scheme $\mathcal{M}(3)$ of semistable rank 2 coherent sheaves with Chern classes $c_1 = 0,\ c_2 = 3,\ c_3 = 0$ on $\mathbb{P}^3$, general points of which correspond to sheaves whose singular loci contain components of dimensions both 0 and 1. These sheaves are produced by elementary transformations of stable reflexive rank 2 sheaves with $c_1 = 0,\ c_2 = 2$ along a disjoint union of a projective line and a collection of points in $\mathbb{P}^3$. The constructed families of sheaves provide first examples of irreducible components of the Gieseker-Maruyama moduli scheme such that their general sheaves have singularities of mixed dimension.

#### 2.1.5 Alexander Kalmynin

[1] On Novák numbers
Sbornik: Mathematics(2018), 209 (4):491

New lower bounds are obtained for the number $\mathcal{N}_B(x)$ of Novák numbers not exceeding the given quantity $x$. In addition, conditioned on the generalized Riemann Hypothesis, upper bounds are found for the number of prime factors of Novák numbers and a description of the prime factors of Novák numbers $N$ such that $2N$ is a Novák-Carmichael number is presented.

[2] Omega-theorems for the Riemann zeta function and its derivatives near the line $\mathrm{Re}\, s = 1$.
Acta Arithmetica 186 (2018), 201-217

We generalize the method of S. P. Zaitsev (2000) in order to prove omega-theorems for the Riemann zeta function and its derivatives in some regions near the line $\mathrm{Re}\, s = 1$.

[3] Large values of short character sums
arXiv:1712.08080 *to appear in Journal of Number Theory*

In this paper, we prove that for any $A > 0$ there exist infinitely many primes $p$ for which sums of the Legendre symbols modulo $p$ over an interval of length $(\ln p)^A$ can take large values.

[4] Intervals between numbers that are sums of two squares
arXiv:1706.07380 *submitted to Mathematika*

In this paper, we improve the moment estimates for the gaps between numbers that can be represented as a sum of two squares of integers. We consider certain sum of Bessel functions and prove the upper bound for its weighted mean value. This bound provides estimates for the $\gamma$-th moments of gaps for all $\gamma \leq 2$.

### 2.1.6 Ilya Kirillov

[1] Morse-Darboux lemma for surfaces with boundary
Journal of Geometry and Physics, 2018, Vol. 129, pp. 34-40.

In this paper we formulate and prove an analog of the classical Morse-Darboux lemma for the case of a surface with boundary.

### 2.1.7 Dmitry Koshelev

[1] Non-split toric codes
https://www.researchgate.net/publication/325020154_Non-split_toric_codes, *to appear in Problems of Information Transmission.*

In the article we introduce a new wide class of error-correcting codes called non-split toric codes. These codes are a natural generalization of toric codes, where non-split algebraic tori are taken instead of usual (i.e., split) ones. The main advantage of new codes is their cyclicity and hence they possibly can be decoded quite quickly. Many classical codes such as (doubly-extended) Reed-Solomon and (projective) Reed-Muller codes are contained (up to an equivalence) in the new class. Our codes are explicitly described in terms of algebraic and toric geometries over finite fields, therefore they can be easily constructed in the practice. Finally, we produce new cyclic reversible codes, which are non-split toric ones on the del Pezzo surface of degree 6 and Picard number 1. We also compute their parameters, which prove to attain current lower bounds at least for small finite fields.

### 2.1.8 Konstantin Loginov

[1] Standard models of degree 1 del Pezzo fibrations.

We construct a standard birational model (a model that has Gorenstein canonical singularities) for the three-dimensional del Pezzo fibrations $\pi : X \longrightarrow C$ of degree 1 and relative Picard number 1. We also embed the standard model into the relative weighted projective space $\mathbb{P}_C(1,1,2,3)$. Our construction works in the $G$-equivariant category where $G$ is a finite group.

To appear in Moscow Mathematical Journal.

[2] On non-rational fibers of del Pezzo fibrations over curves.

We consider threefold del Pezzo fibrations over a curve germ whose central fiber is non-rational. Under the additional assumption that the singularities of the total space are at worst ordinary double points, we apply a suitable base change and show that there is a 1-to-1 correpspondence between such fibrations and certain non-singular del Pezzo fibrations equipped with a cyclic group action.

arXiv preprint, 1811.04418.

### 2.1.9 Pavel Osipov

[1] Projective Hessian and Sasakian manifolds
arXiv:arXiv:1803.02799v2 *submitted to Transformation Groups*

The Hessian geometry is the real analogue of the Kähler one. Sasakian geometry is an odd-dimensional counterpart of the Kähler geometry. In the paper, we study the connection between projective Hessian and Sasakian manifolds analogous to the one between Hessian and Kähler manifolds. In particular, we construct a Sasakian structure on a product of tangent bundle to any Hessian manifold amd real line. Especially, we are interested in the case of invariant structure on Lie groups. We define semi-Sasakian Lie groups as a generalization of Sasakian Lie groups. Then we construct a semi-Sasakian structure on some group. Further, we describe examples of homogeneous Hessian Lie groups and corresponding semi-Sasakian Lie groups. The big class of projective Hessian Lie groups can be constructed by homogeneous regular domains in an affine space. The groups SO(2) and SU(2) belong to another kind of examples. Using them, we construct semi-Sasakian structures on the group of the Euclidean motions of the real plane and the group of isometries of the complex plane.

## 2.2 Scientific conferences and seminar talks

### 2.2.1 Semeyon Abramyan

[1] Conference "International Seminar on Toric Topology and Homotopy Theory", Steklov Mathematical Institite, Moscow, Russia, May 31–June 1

Talk "Iterated higher Whitehead products in topology of moment-angle complexes"

[2] Conference "Glances@Manifolds", Jagiellonian University, Krakw, Poland, July 2–6, 2018

[3] Conference "Young Topologists Meeting 2018", University of Copenhagen, Copenhagen, Denmark, July 9–13

[4] Conference "Siberian summer school: Current developments in Geometry", Sobolev Institute of Mathematics, Novosibirsk, Russia, August 27–September 1

Talk "Higher Whitehead products in toric topology"

[5] Course "Rational Homotopy Theory", Independent University of Moscow, Moscow, Russia, Fall 2018, Lecturer

[6] Talk "Rational Homotopy Type" at "Geometry and Topology" seminar (Moscow State University)

[7] Talk "Formality of Kähler Manifold" at "Geometry and Topology" seminar (Moscow State University)

[8] Talk "On Pontryagin Algebra of a Loop Space" at "Geometry and Topology" seminar (Moscow State University)

[9] Talk "Real Bott Periodicity" at "Homotopy Theory" seminar (Higher School of Ecomomics)

### 2.2.2 Nikolay Bogachev

[1] Conference "Automorphic Forms and Algebraic Geometry", Steklov Mathematical Institute (St. Petersburg branch), St. Petersburg, Russia. May, 20 — May, 25

Talk "On reflective hyperbolic lattices"

[2] Seminar "Automorphic Forms and Applications" of the Lab of Algebraic Geometry, Higher School of Economics, Moscow, Russia, February.

Talk "On methods of classification of reflective hyperbolic lattices"

[3] Seminar “Geometry and Topology”, Moscow State University, Moscow, Russia, March.

Talk “Discrete Groups and Reflection Groups”

#### 2.2.3 Alexei Ilyin

[1] Conference “Representation Theory, Geometry, and Quantization: The Mathematical Legacy of Bertram Kostant”, Boston, USA, May, 28 – June, 1

[2] Conference “Interactions of quantum affine algebras with cluster algebras, current algebras and categorification”, Washington D.C, USA, June, 2 – June, 8

Talk “Degeneration of Bethe subalgebras in the Yangians”

[3] Summer School on Geometric Representation Theory, IST Austria, July, 9 – July, 13

#### 2.2.4 Alexei Ivanov

[1] Conference “Local and Nonlocal Geometry of PDEs and Integrability”, Trieste (Italy), October, 8 – October, 12

Talk “Complex invariant Einstein metrics on flag manifolds with T-root system $BC_2$”

[2] Conference “Differential Equations and Related Questions of Mathematics”, Kolomna, June, 15 – June, 16

Talk “Complex invariant Einstein metrics on flag manifolds”

#### 2.2.5 Alexander Kalmynin

[1] 6th International Conference on Uniform Distribution Theory, Luminy, France, October 1–5, 2018

Talk “Large values of short character sums”

[2] School and research conference “Modular forms and beyond”, St. Petersburg, Russia, May 21–26, 2018

Talk “Cohen-Kuznetsov series and intervals between numbers that are sums of two squares”

[3] XV International Conference Algebra, Number Theory and Discrete Geometry: modern problems and applications, dedicated to the centenary of the birth of the Doctor of Physical and Mathematical Sciences, Professor of the Moscow State University Korobov Nikolai Mikhailovich, Tula, Russia, May 28–31, 2018

Talk “Intervals between numbers which can be expressed as sum of two squares”

[4] International conference ”Algebra, algebraic geometry and number theory” dedicated to the memory of academician Igor Rostislavovich Shafarevich, Moscow, Russia, June 13–14, 2018

Talk “Large values of short character sums”

[5] Alexei Zykin memorial conference, Moscow, Russia, June 21, 2018

Talk “Large values of short character sums”

[6] Seminar “Moscow– Saint-Petersburg”, Moscow, Russia, November 15-16, 2018

Talk “Distribution of quadratic residues”

[7] Seminar “Automorphic forms and their applications”, Moscow, Russia, March 5 and 12, 2018

Talks “Mock theta functions”

[8] Seminar “Functional analysis and noncommutative geometry”, Moscow, Russia, March 12 and 26, 2018

Talks “Noncommutative $L^p$-spaces

[9] Seminar “Contemporary problems in number theory”, Moscow, Russia, December 13 and 20, 2018

Talks “Erdős discrepancy problem”

#### 2.2.6 Ilya Kirillov

[1] Conference “Geometrietag 2018 in Dresden”, Dresden, December, 7 – 9

Talk “Coadjoint orbits of symplectic diffeomorphisms of surfaces with boundary”

[2] Seminar “Modern Geometric Methods” at MSU, Moscow, April, 22

Talk “Coadjoint orbits of symplectic diffeomorphisms of surfaces with boundary”

[3] Research Internship at FSU under supervision of prof. Matveev, Jena, November, 22 – December, 22

[4] Research Program FUSRP 2018 at Fields Institute under supervision of prof. Zhu, Toronto, June, 2 – August, 31

#### 2.2.7 Dmitry Koshelev

[1] Talk “On rationality of Kummer surfaces over the field of two elements and the discrete logarithm problem” at “Iskovskikh Seminar” (Steklov Mathematical Institute, May)

[2] Talk “Non-split toric codes” at “Interdepartmental Seminar on Discrete Mathematics” (Moscow Institute of Physics and Technology, November), “Seminar on Coding Theory” (Institute for Information Transmission Problems, November), and “Iskovskikh Seminar” (Steklov Mathematical Institute, December)

#### 2.2.8 Konstantin Loginov

[1] Workshop on birational geometry, Laboratory of Algebraic Geometry, Higher School of Economics, Moscow, October 29–31, 2018

[2] Conference “Lie groups and Lie algebras”, Samara State University, August 18–26

[3] Siberian Summer School “Current developments in Geometry”, Laboratory of mirror symmetry and automorphic forms, Higher School of Economics, Novosibirsk, August 27—September 1, 2018

[4] Seminar “Moscow – Saint-Petersburg”, Laboratory of mirror symmetry and automorphic forms, Higher School of Economics, 15–16 November, 2018

[5] Talk “Non-rational fibers of del Pezzo fibrations” at Iskovskikh Seminar (Steklov Mathematical Institute)

[6] Talk “Kawamata-Viehweg vanishing theorem” at Student Geometric Seminar (Independent University of Moscow)

#### 2.2.9 Pavel Osipov

[1] Talk “Hessian manifolds” at “Geometric Structures on Manifolds” (High School of Economics), March, 8

[2] Talk “Hessian manifolds” at “Riemann surfaces, Lie algebras and mathematical physics” (Independent Moscow University ), April 27

[3] Talk “Flat affine manifolds” at “Master’s Seminar” (High School of Economics), May, 12

[4] Talk “The Collar Theorem” at “Differential geometry, smooth structures, and gauge theory” (High School of Economics), May, 22

# 3 Program: Simons IUM fellowships

## 3.1 Research

#### 3.1.1 Alexander Belavin

[1] ”Special Geometry on Calabi-Yau Moduli Spaces and Q-Invariant Milnor Rings”, Proc. Int. Congress of Math. 2018 (ICM2018), Rio de Janeiro, Brazil, 1-9 August 2018, Vol. 2, p.2553-2566 (2018); arXiv:1808.05470.

In this paper the special Kähler manifolds that are the moduli spaces of Calabi–Yau (CY) manifolds are considered . The special Kähler geometry determines the low-energy effective theory which arises in Superstring theory after the compactification on a CY manifold. For the cases, where the CY manifold is given as a hypersurface in the weighted projective space, a new procedure for computing the Kähler potential of the moduli space has been proposed by us recently. The method is based on the fact that the moduli space of CY manifolds is a marginal subspace of the Frobenius manifold which arises on the deformation space of the corresponding Landau–Ginzburg superpotential. I review this approach and demonstrate its efficiency by computing the Special geometry of the 101-dimensional moduli space of the quintic threefold around the orbifold point.

[2] With K. Aleshkin, "Exact computation of the Special geometry for Calabi-Yau hypersurfaces of Fermat type",
Pisma v ZhETF, 108(10), 723-724 (2018)22:57 17.11.2018.

In this paper we continue to develop our method for effectively computating the special K?hler geometry on the moduli space of Calabi-Yau manifolds. We generalize it to all polynomial deformations of Fermat hypersurfaces.

[3] With K. Aleshkin and A. Litvinov.
"Two-sphere partition functions
and Kähler potentials on CY moduli spaces".
Pisma v ZhETF, 108(10), 725 (2018)22:57 17.11.2018.

In this paper we study the relation between exact partition functions of gauged $N = (2, 2)$ linear sigma-models on $S^2$ and Kähler potentials of CY manifolds proposed by Jockers et all. We suggest to use a mirror version of this relation. For a class of manifolds given by a Fermat hypersurfaces in weighted projective space we check the relation by explicit calculation.

#### 3.1.2 Yurii Burman

[1] Higher matrix-tree theorems and Bernardi polynomial
To be published in J. of Algebraic Combinatorics, 2018.

The classical matrix-tree theorem discovered by G. Kirchhoff in 1847 expresses the principal minor of the $n \times n$ Laplace matrix as a sum of monomials of matrix elements indexed by directed trees with $n$ vertices. We prove, for any $k \geq n$, a three-parameter family of identities between degree $k$ polynomials of matrix elements of the Laplace matrix. For $k = n$ and special values of the parameters the identity turns to be the matrix-tree theorem.

For the same values of parameters and arbitrary $k \geq n$ the left-hand side of the identity becomes a specific polynomial of the matrix elements called higher determinant of the matrix. We study properties of the higher determinants; in particular, they have an application (due to M. Polyak) in the topology of 3-manifolds.

#### 3.1.3 Alexei Elagin

[1] (with V.Lunts and O.Schnuerer) "Smoothness of Derived Categories of Algebras", https://arxiv.org/abs/1810.07626

We prove smoothness in the dg sense of the bounded derived category of finitely generated modules over any finite-dimensional algebra over a perfect field, hereby answering a question of Iyama. More generally, we prove this statement for any algebra over a perfect field that is finite over its center and whose center is finitely generated as an algebra. These results are deduced from a general sufficient criterion for smoothness.

### 3.1.4 Konstantin Fedorovsky

[1] With Yu. Belov
Model spaces containing univalent functions
Russian Math. Surveys, 2018, Vol. 73, No. 1, pp. 172–174

Motivated by a problem in approximation theory, we find a necessary and sufficient condition for a model (backward shift invariant) subspace $K_\Theta = H^2 \ominus \Theta H^2$ of the Hardy space $H^2$ to contain a bounded univalent functions.

[2] With A. Bagapsh
$C^m$ approximation on functions by solutions of second-order elliptic systems on compact sets in the plane.
Proc. Steklov Inst. Math., 2018. Vol. 301, pp. 7–17.

This paper is a brief survey of recent results in problems of approximating functions by solutions of homogeneous elliptic systems of partial differential equations on compact sets in the plane in norms of the spaces $C^m$, $m \geq 0$. We are focused on second-order systems. For such systems this paper complements the recent survey by K. Fedorovskiy, M. Mazalov and P. Paramonov ("Russian Math. Surveys", 2012), where the problems of $C^m$-approximation of functions by holomorphic, harmonic and polyanalytic functions, and by solutions of homogeneous elliptic PDE with constant complex coefficients are considered.

[3] Two problems on approximation by solutions of elliptic systems on compact sets in the plane.
Compl. Var. Ellipt. Eq., 2018, Vol. 63, No. 7–8, pp. 961–975.

Motivated by recent results by M. Mazalov about uniform approximation of functions by solutions of elliptic equations with constant complex coefficients we study two problems on approximation of functions by solutions of general homogeneous elliptic second-order systems of partial differential equations. The approximation is considered in spaces of continuous and $C^1$-functions on compact sets in the complex plane.

[4] With E. Abakumov
Analytic balayage of measures, Carathodory domains, and badly approximable functions in $L^p$

C. R. Acad. Sci. Paris, Ser. I, 2018, Vol. 356, pp. 870-874

We give new formulae for analytic balayage of measures supported on subsets of Carathéodory compact sets in the complex plane, and consider a related problem of description of badly approximable functions in $L^p(\mathbb{T})$.

[5] Carathéodory sets and analytic balayage of measures
Sb. Math., 2018, Vol. 209, No. 9, pp. 1376–1389

We consider the concept of an analytic balayage of measures introduced by D. Khavinson. New formulae for analytic balayage are obtained in the case when the support of a measure lies inside some Carathéodory compact set, and balayage onto its boundary is considered. The constructions are based on recent results on the boundary behaviour of conformal mappings of the unit disc onto Carathéodory domains.

[6] With Yu. Belov and A. Borichev
Nevanlinna domains with large boundaries
arXiv:1808.07436, *submitted to Journal of Functional Analysis*

We establish the existence of Nevanlinna domains with large boundaries. In particular, these domains can have boundaries of positive planar measure. The sets of accessible points can be of any Hausdorff dimension between 1 and 2. As a quantitative counterpart of these results, we construct rational functions univalent in the unit disc with extremely long boundaries for a given amount of poles.

#### 3.1.5 Anton Fonarev

[1] On the bounded derived category of $\mathsf{IGr}(3, 7)$
arXiv:1804.06946 *submitted to International Mathematics Research Notices*

We construct a mininal Lefschetz decomposition of the bounded derived category of coherent sheaves on the isotropic Grassmannian $\mathsf{IGr}(3, 7)$. Moreover, we show that $\mathsf{IGr}(3, 7)$ admits a full exceptional collection consisting of equivariant vector bundles.

#### 3.1.6 Alexander Kolesnikov

[1] Mass transportation functionals on the sphere with applications to the logarithmic Minkowski problem
arXiv: 1807.07002 *Submitted to Moscow Mathematical Journal*

We study the transportation problem on the unit sphere $S^{n-1}$ for symmetric probability measures and the cost function $c(x, y) = \log \frac{1}{\langle x,y\rangle}$. We calculate the variation of the corresponding Kantorovich functional $K$ and study a naturally associated metric-measure space on $S^{n-1}$ endowed with a Riemannian metric generated by the corresponding transportational potential. We introduce a new transportational functional which minimizers are solutions to the symmetric log-Minkowski problem and prove that $K$ satisfies the following analog of the Gaussian transportation inequality for the uniform probability measure $\sigma$ on $S^{n-1}$: $\frac{1}{n}Ent(\nu) \geq K(\sigma, \nu)$. It is shown that there exists a remarkable similarity between our results and the theory of the Kähler–Einstein equation on Euclidean space. As a by-product we obtain a new proof of uniqueness of solution to the log-Minkowski problem for the uniform measure.

[2] With Galyna V. Livshyts

On the Gardner–Zvavitch conjecture: symmetry in the inequalities of Brunn–Minkowski type

arXiv: 1807.06952, *Submitted to Advances in Mathematics*

In this paper, we study the conjecture of Gardner and Zvavitch from [**?**], which suggests that the standard Gaussian measure $\gamma$ enjoys $\frac{1}{n}$-concavity with respect to the Minkowski addition of **symmetric** convex sets. We prove this fact up to a factor of 2: that is, we show that for symmetric convex $K$ and $L$,

$$\gamma(\lambda K + (1-\lambda)L)^{\frac{1}{2n}} \geq \lambda\gamma(K)^{\frac{1}{2n}} + (1-\lambda)\gamma(L)^{\frac{1}{2n}}.$$

Further, we show that under suitable dimension-free uniform bounds on the Hessian of the potential, the log-concavity of even measures can be strengthened to $p$-concavity, with $p > 0$, with respect to the addition of symmetric convex sets.

#### 3.1.7 Mikhail Lashkevich

[1] With Ya. Pugai

The complex sinh-Gordon model: form factors of descendant operators and current–current perturbations

arXiv:1811.02631, submitted to JHEP.

We study quasilocal operators in the quantum complex sinh-Gordon theory in the form factor approach. The free field procedure for descendant operators is developed by introducing the algebra of screening currents and related algebraic objects. We work out null vector equations in the space of operators. Further we apply the proposed algebraic structures to constructing form factors of the conserved currents $T_k$ and $\Theta_k$. We propose also form factors of current–current operators of the form $T_kT_{-l}$. Explicit computations of the four-particle form factors allow us to verify the recent conjecture of Smirnov and Zamolodchikov about the structure of the exact scattering matrix of an integrable theory perturbed by a combination of irrelevant operators. Our calculations confirm that such perturbations of the complex sinh-Gordon model and of the $\mathbb{Z}_N$ symmetric Ising models result in extra CDD factors in the $S$ matrix.

### 3.1.8 Taras Panov

[1] With Ivan Limonchenko and Zhi Lu.
Calabi–Yau hypersurfaces and SU-bordism.
Trudy Matematicheskogo Instituta imeni V. A. Steklova, 2018, Vol. 302, pp. 287–295 (Russian); Proceedings of the Steklov Institute of Mathematics, 2018, Vol. 302, pp. 270–278 (English).

Batyrev constructed a family of Calabi–Yau hypersurfaces dual to the first Chern class in toric Fano varieties. Using this construction, we introduce a family of Calabi–Yau manifolds whose SU-bordism classes generate the special unitary bordism ring $\Omega^{SU}[\frac{1}{2}] \cong \mathbb{Z}[\frac{1}{2}][y_i \colon i \geq 2]$. We also describe explicit Calabi–Yau representatives for multiplicative generators of the $SU$-bordism ring in low dimensions.

[2] With Stephen Theriault.
The homotopy theory of polyhedral products associated with flag complexes.
Compositio Math., 2019, Vol. 155, No. 1, pp. 206–228;
DOI:10.1112/S0010437X18007613

If $K$ is a simplicial complex on $m$ vertices the flagification of $K$ is the minimal flag complex $K^f$ on the same vertex set that contains $K$. Letting $L$ be the set of vertices, there is a sequence of simplicial inclusions $L \to K \to K^f$. This induces a sequence of maps of polyhedral products $(X, A)^L \xrightarrow{g} (X, A)^K \xrightarrow{f} (X, A)^{K^f}$. We show that $\Omega f$ and $\Omega f \circ \Omega g$ have right homotopy inverses and draw consequences. We also show that for flag $K$ the polyhedral product of the form $(CY, Y)^K$ is a co-$H$-space if and only if the 1-skeleton of $K$ is a chordal graph, and deduce that the maps $f$ and $f \circ g$ have right homotopy inverses in this case.

[3] With Hiroaki Ishida and Roman Krutowski.
Basic cohomology of canonical holomorphic foliations on complex moment-angle manifolds
arXiv:1811.12038

Battaglia and Zaffran computed the basic Betti numbers for the canonical holomorphic foliation on a moment-angle manifold corresponding to a shellable fan. They conjectured that the basic cohomology ring in the case of any complete simplicial fan has a description similar to the cohomology ring of a complete simplicial toric variety due to Danilov and Jurkiewicz. In this work we prove the conjecture. The proof uses an Eilenberg-Moore spectral sequence argument; the key ingredient is the formality of the Cartan model for the torus action on a moment-angle manifold.

### 3.1.9 Alexei Penskoi

[1] With N. S. Nadirashvili

An isoperimetric inequality for the second non-zero eigenvalue of the Laplacian on the projective plane

Geom. Funct. Anal., 2018, Vol. 28, No. 5, pp. 1368-1393.

In this paper we prove an isoperimetric inequality for the second non-zero eigenvalue of the Laplace-Beltrami operator on the real projective plane. For a metric of unit area this eigenvalue is not greater than $20\pi$. This value is attained in the limit by a sequence of metrics of area one on the projective plane. The limiting metric is singular and could be realized as a union of the projective plane and the sphere touching at a point, with standard metrics and the ratio of the areas $3:2$. It is also proven that the multiplicity of the second non-zero eigenvalue on the projective plane is at most 6.

[2] Isoperimetric inequalities for higher eigenvalues of Laplace-Beltrami operator on surfaces

To appear in Proceedings of the Steklov Institute of Mathematics, 2019, Vol. 305.

In this paper recent advances in isoperimetric inequalities for higher eigenvalues of the Laplace-Beltrami operator on the sphere and on the projective plane are discussed.

[3] With M. A. Karpukhin, N. S. Nadirashvili and I. V. Polterovich

An isoperimetric inequality for Laplace eigenvalues on the sphere.

arXiv:1706.05713, *submitted to Journal of Differential Geometry, revisions are required and submitted*

In this paper we show that for any positive integer $k$, the $k$-th nonzero eigenvalue of the Laplace-Beltrami operator on the two-dimensional sphere endowed with a Riemannian metric of unit area, is maximized in the limit by a sequence of metrics converging to a union of $k$ touching identical round spheres. This proves a conjecture posed by the second author in 2002 and yields a sharp isoperimetric inequality for all nonzero eigenvalues of the Laplacian on a sphere. Earlier, the result was known only for $k = 1$ (J. Hersch, 1970), $k = 2$ (N. Nadirashvili, 2002; R. Petrides, 2014) and $k = 3$ (N. Nadirashvili and Y. Sire, 2017). In particular, we argue that for any $k \geq 2$, the supremum of the $k$-th nonzero eigenvalue on a sphere of unit area is not attained in the class of Riemannian metrics which are smooth outsitde a finite set of conical singularities. The proof uses certain properties of harmonic maps between spheres, the key new ingredient being a bound on the harmonic degree of a harmonic map into a sphere obtained by N. Ejiri.

### 3.1.10 Vladimir Poberezhnyi

[1] With G.F.Helminck and S.V.Polenkova
A geometric construction of solutions of the strict dKP($\Lambda_0$)
J.Geom.Phys., 131 (2018, 189-203)

We associated to each invertible constant pseudo difference operator $\Lambda_0$ of degree one, two integrable hierarchies in the algebra of pseudo difference operators PS$\Delta$, the so-called dKP($\Lambda_0$) hierarchy and its strict version. We show here first that both hierarchies can be described as the compatibility conditions for a proper linearization. Next we present a geometric framework for the construction of solutions of the hierarchies, i.e. we associate to each hierarchy an infinite dimensional variety such that to each point of the variety one can construct a solution of the corresponding hierarchy. This yields a SegalWilson type framework for all these integrable hierarchies.

### 3.1.11 Leonid Rybnikov

[1] Leonid Rybnikov. Cactus Group and Monodromy of Bethe Vectors. *International Mathematics Research Notices*. 2018. No. 1. P. 202-235.

Cactus group is the fundamental group of the real locus of the Deligne-Mumford moduli space of stable rational curves. This group appears naturally as an analog of the braid group in coboundary monoidal categories. We define an action of the cactus group on the set of Bethe vectors of the Gaudin magnet chain corresponding to arbitrary semisimple Lie algebra g. Cactus group appears in our construction as a subgroup in the Galois group of Bethe Ansatz equations. Following the idea of Pavel Etingof, we conjecture that this action is isomorphic to the action of the cactus group on the tensor product of crystals coming from the general coboundary category formalism. We prove this conjecture in the case $\mathfrak{g} = \mathfrak{sl}_2$ (in fact, for this case the conjecture almost immediately follows from the results of Varchenko on asymptotic solutions of the KZ equation and crystal bases). We also present some conjectures generalizing this result to Bethe vectors of shift of argument subalgebras and relating the cactus group with the Berenstein-Kirillov group of piecewise-linear symmetries of the Gelfand-Tsetlin polytope.

[2] With Aleksei Ilin
Degeneration of Bethe subalgebras in the Yangian of $\mathfrak{gl}_n$. *Letters in Mathematical Physics*. 2018. Vol. 108. No. 4. P. 1083-1107.

We study degenerations of Bethe subalgebras $B(C)$ in the Yangian $Y(\mathfrak{gl}_n)$, where $C$ is a regular diagonal matrix. We show that closure of the parameter space of the family of Bethe subalgebras, which parametrizes all possible degenerations, is the Deligne-Mumford moduli space of stable rational curves $\overline{M_{0,n+2}}$. All subalgebras corresponding to the points of $\overline{M_{0,n+2}}$ are free and maximal commutative. We describe explicitly the "simplest" degenerations and show that every degeneration is the composition of the simplest ones. The Deligne-Mumford space $\overline{M_{0,n+2}}$ generalizes to other root systems as some De Concini-Procesi resolution of some toric variety. We state a conjecture generalizing our results to Bethe subalgebras in the Yangian of arbitrary simple Lie algebra in terms of this De Concini-Procesi resolution.

[3] With Michael Finkelberg, Kamnitzer J., Pham K., Weekes A.
Comultiplication for shifted Yangians and quantum open Toda lattice *Advances in Mathematics.* 2018. Vol. 327. P. 349-389.

We study a coproduct in type A quantum open Toda lattice in terms of a coproduct in the shifted Yangian of $sl_2$. At the classical level this corresponds to the multiplication of scattering matrices of euclidean $SU(2)$ monopoles. We also study coproducts for shifted Yangians for any simply-laced Lie algebra.

[4] With Michael Finkelberg, Alexander Kuznetsov, Galyna Dobrovolska. Towards a cluster structure on trigonometric zastava *Selecta Mathematica, New Series.* 2018. Vol. 24. No. 1. P. 187-225.

We study a moduli problem on a nodal curve of arithmetic genus 1, whose solution is an open subscheme in the zastava space for projective line. This moduli space is equipped with a natural Poisson structure, and we compute it in a natural coordinate system. We compare this Poisson structure with the trigonometric Poisson structure on the transversal slices in an affine flag variety. We conjecture that certain generalized minors give rise to a cluster structure on the trigonometric zastava.

[5] A proof of the Gaudin Bethe Ansatz conjecture. *International Mathematics Research Notices*, published online, DOI 10.1093/imrn/rny245

The Gaudin algebra is the commutative subalgebra in $U(\mathfrak{g})^{\otimes N}$ generated by higher integrals of the quantum Gaudin magnet chain attached to a semisimple Lie algebra $\mathfrak{g}$. This algebra depends on a collection of pairwise distinct complex numbers $z_1, \ldots, z_N$. We prove that this subalgebra has a cyclic vector in the space of singular vectors of the tensor product of any finite-dimensional irreducible $\mathfrak{g}$-modules, for all values of the parameters $z_1, \ldots, z_N$. We deduce from this result the Bethe Ansatz conjecture in the Feigin-Frenkel form which states that the joint eigenvalues of the higher Gaudin Hamiltonians on the tensor product of irreducible finite-dimensional $\mathfrak{g}$-modules are in 1-1 correspondence with monodromy-free ${}^L G$-opers on the projective line with regular singularities at the points $z_1, \ldots, z_N, \infty$ and the prescribed residues at the singular points.

[6] With Aleksei Ilin
Bethe subalgebras in Yangians and the wonderful compactification, arXiv:1810.07308

We study the family of Bethe subalgebras in the Yangian $Y(\mathfrak{g})$ parameterized by the corresponding adjoint Lie group $G$. We describe their classical limits as subalgebras in the algebra of polynomial functions on the formal Lie group $G_1[[t^{-1}]]$. In particular we show that, for regular values of the parameter, these subalgebras are free polynomial algebras with the same Poincare series as the Cartan subalgebra of the Yangian. Next, we extend the family of Bethe subalgebras to the De Concini–Procesi wonderful compactification $\overline{G} \supset G$ and describe the subalgebras corresponding to generic points of any stratum in $\overline{G}$ as Bethe subalgebras in the Yangian of the corresponding Levi subalgebra in $\mathfrak{g}$. In particular, we describe explicitly all Behte subalgebras corresponding to the closure of the maximal torus in the wonderful compactification.

[7] With Mikhail Zavalin
Gelfand-Tsetlin degeneration of shift of argument subalgebras in types B and C. arXiv:1807.11126

The universal enveloping algebra of any semisimple Lie algebra $\mathfrak{g}$ contains a family of maximal commutative subalgebras, called shift of argument subalgebras, parametrized by regular Cartan elements of $\mathfrak{g}$. For $\mathfrak{g} = \mathfrak{gl}_n$ the Gelfand-Tsetlin commutative subalgebra in $U(\mathfrak{g})$ arises as some limit of subalgebras from this family. We study the analogous limit of shift of argument subalgebras for the Lie algebras $\mathfrak{g} = \mathfrak{sp}_{2n}$ and $\mathfrak{g} = \mathfrak{so}_{2n+1}$. The limit subalgebra is described explicitly in terms of Bethe subalgebras in twisted Yangians $Y^-(2)$ and $Y^+(2)$, respectively. We index the eigenbasis of such limit subalgebra in any irreducible finite-dimensional representation of $\mathfrak{g}$ by Gelfand-Tsetlin patterns of the corresponding type, and conjecture that this indexing is, in appropriate sense, natural. According to Halacheva-Kamnitzer-Rybnikov-Weekes such eigenbasis has a natural $\mathfrak{g}$-crystal structure. We conjecture that this crystal structure coincides with that on Gelfand-Tsetlin patterns defined by Littelmann.

[8] With Mikhail Zavalin
Gelfand-Tsetlin degeneration of shift of argument subalgebras in type D. arXiv:1810.06763

The universal enveloping algebra of any semisimple Lie algebra $\mathfrak{g}$ contains a family of maximal commutative subalgebras, called shift of argument subalgebras, parametrized by regular Cartan elements of $\mathfrak{g}$. For $\mathfrak{g} = \mathfrak{gl}_n$ the Gelfand-Tsetlin commutative subalgebra in $U(\mathfrak{g})$ arises as some limit of subalgebras from this family. In our previous work [**?**] we studied the analogous limit of shift of argument subalgebras for the Lie algebras $\mathfrak{g} = \mathfrak{sp}_{2n}$ and $\mathfrak{g} = \mathfrak{o}_{2n+1}$. We described the limit subalgebras in terms of Bethe subalgebras of twisted Yangians $Y^-(2)$ and $Y^+(2)$, respectively, and parametrized the eigenbases of these limit subalgebras in the finite dimensional irreducible highest weight representations by Gelfand-Tsetlin patterns of types C and B. In this note we state and prove similar results for the last case of classical Lie algebras, $\mathfrak{g} = \mathfrak{o}_{2n}$. We describe the limit shift of argument subalgebra in terms of the Bethe subalgebra in the twisted Yangian $Y^+(2)$ and give a natural indexing of its eigenbasis in any finite dimensional irreducible highest weight $\mathfrak{g}$-module by type D Gelfand-Tsetlin patterns.

#### 3.1.12 George Shabat

[1] Counting Belyi pairs over finite fields, In 2016 MATRIX Annals (eds, David R. Wood, Jan de Gier, Cheryl E. Praeger, Terence Tao). MATRIX Book Series, Volume 1, p. 305-322, 2018.

The techniques of counting Belyi pairs (which in the zero characteristic is equivalent to counting dessins d'enfance, an active field of research during the last decades) is extended to the positive characteristic.

[2] The economy in the languages of science and in the natural language (joint with G.E. Kreydlin, in russian). Economy in the language and communication (ed., L.L. Fedorova). Moscow, RSUH, p. 27-37, 2018.

Some specific features of the languages of science (especially mathematics) are highlighted, that allow the compact and precise forms of the statements.

[3] Computer experiment in teaching mathematics (in russian). Proceedings of the IV International Conference "Current Problems of teaching mathematics and computer science in the schools and universities". Moscow, MPGU, 2018. p. 253-257.

The possibilities of modernization of teaching mathematics based on the regular computer experiments are considered. The idea of replacing the memorization of statements and formulas by the verifiable facts is formulated.

[4] Belyi pairs in the critical filtrations of Hurwitz spaces. To appear in the Nankai Series in Pure, Applied Mathematics and Theoretical Physics, published by the World Scientific Company.

The stratification of the Hurwitz and the moduli spaces are introduced, the Belyi pairs constituting the zero-dimensional part of it.

#### 3.1.13 Stanislav Shaposhnikov

[1] Bogachev V.I., Krasovitskii T.I., Shaposhnikov S.V. On non-uniqueness of probability solutions to the two-dimensional stationary Fokker–Planck–Kolmogorov equation. Doklady Mathematics, 2018, V. 98, N. 2, P. 475–479.

In this paper the problem of uniqueness of probability solutions to the two-dimensional stationary Fokker–Planck–Kolmogorov equation is considered. Under broad conditions, it is proved that the existence of two different probability solutions implies the existence of an infinite set of linearly independent probability solutions.

[2] Bogachev V.I., Rckner M., Shaposhnikov S.V. Convergence to stationary measures in nonlinear Fokker–Planck–Kolmogorov equations. Doklady Mathematics, 2018, V. 98, N. 2, P. 452-457.

In this paper the convergence of solutions of nonlinear Fokker–Planck–Kolmogorov equations to stationary solutions is studied. Broad sufficient conditions for convergence in variation with an exponential bound are obtained.

[3] Bogachev V.I., Rckner M., Shaposhnikov S.V. The Poisson equation and estimates for distances between stationary distributions of diffusions. Journal of Mathematical Sciences, 2018, V. 232, N. 3, P. 254–282.

In this paper we estimate distances between stationary solutions to Fokker–Planck–Kolmogorov equations with different diffusion and drift coefficients. To this end we study the Poisson equation on the whole space. We have obtained sufficient conditions for stationary solutions to satisfy the Poincare and logarithmic Sobolev inequalities.

#### 3.1.14 Mikhail Skopenkov

[1] Elements of mathematics in problems. Through circles and olympiads to profession. Ed. by A. Skopenkov, M. Skopenkov, and A. Zaslavskiy. Moscow Center for Continuous Mathematical Education, 2018, 592 pp. ISBN 978-5-4439-1239-4 (in Russian).

A problem-based introduction to the subjects of mathematics traditionally studied in circles, with vast coverage of those subjects.

[2] Skopenkov M., Krasauskas R., Surfaces containing two circles through each point, Math. Ann. (2018), doi:10.1007/s00208-018-1739-z, Fulltext available at: https://rdcu.be/4rXh.

We find all analytic surfaces in space $\mathbb{R}^3$ such that through each point of the surface one can draw two transversal circular arcs fully contained in the surface. The problem of finding such surfaces traces back to the works of Darboux from XIXth century. We prove that such a surface is an image of a subset of one of the following sets under some composition of inversions:

- the set $\{\, p+q : p \in \alpha, q \in \beta \,\}$, where $\alpha, \beta$ are two circles in $\mathbb{R}^3$;
- the set $\{\, 2\frac{[p\times q]}{|p+q|^2} : p \in \alpha, q \in \beta, p+q \neq 0 \,\}$, where $\alpha, \beta$ are two circles in $S^2$;
- the set $\{\, (x,y,z) : Q(x,y,z,x^2+y^2+z^2) = 0 \,\}$, where $Q \in \mathbb{R}[x,y,z,t]$ has degree 2 or 1.

The proof uses a new factorization technique for quaternionic polynomials.

[3] M. Skopenkov, Discrete field theory: symmetries and conservation laws, https://arxiv.org/abs/1709.0 submitted (new version prepared in 2018).

We present a general algorithm constructing a discretization of a classical field theory from a Lagrangian. We prove a discrete Noether theorem relating symmetries to conservation laws and an energy conservation theorem not based on any symmetry. This gives exact conservation laws for several discrete field theories: electrodynamics, gauge theory, Klein-Gordon and Dirac ones. In particular, we construct a conserved discrete energy-momentum tensor, approximating the continuum one at least for free fields. The theory is stated in topological terms, such as coboundary and products of cochains.

#### 3.1.15 Evgeni Smirnov

[1] Multiple flag varieties

VINITI, Itogi Nauki i Tekhniki. Ser. Sovrem. Mat. i ee pril. Temat. obz., 2018, 147, pp. 84–119. English translation: to appear in Journal of Mathematical Sciences.

This is a survey of results on multiple flag varieties, i.e. varieties of the form $G/P_1 \times \cdots \times G/P_k$. We provide a classification of multiple flag varieties of complexity 0 and 1 and results on the combinatorics and geometry of $B$-orbits and their closures in double cominuscule flag varieties. We also discuss questions of finiteness for the number of $G$-orbits and existence of an open $G$-orbits on a multiple flag variety.

#### 3.1.16 Ilya Vyugin

[1] S.V. Konyagin, S.V. Makarychev, I.E. Shparlinski, I.V. Vyugin, "On the new bound for the number of solutions of polynomial equations in subgroups and the structure of graphs of Markoff triples", submitted to Quarterly Journal of Mathematics.

We sharpen the bounds of J. Bourgain, A. Gamburd and P. Sarnak (2016) on the possible number of nodes outside the "giant component" and on the size of individual connected components in the suitably defined functional graph of Markoff triples modulo $p$. This is a step towards the conjecture that there are no such nodes at all. These results are based on some new ingredients and in particular on a new bound of the number of solutions of polynomial equations in cosets of multiplicative subgroups in finite fields, which generalises previous results of P. Corvaja and U. Zannier (2013).

[2] I.V. Vyugin, "On the Bound of Inverse Images of a Polynomial Map" preprint, 2018.

Let $f_1(x), \ldots, f_n(x)$ be some polynomials. The upper bound on the number of $x \in \mathbb{F}_p$ such that $f_1(x), \ldots, f_n(x)$ are roots of unit of order $t$ is obtained. This bound generalize the known bound for linear polynomials $f_1(x), \ldots, f_n(x)$ to the case of polynomials of degrees greater than one. The bound is obtained over fields of positive characteristic and over the complex field.

### 3.1.17 Vladimir Zhgoon

[1] V. P. Platonov, V. S. Zhgoon, G. V. Fedorov On the Periodicity of Continued Fractions in Hyperelliptic Fields over Quadratic Constant Field

Doklady Mathematics, 2018, 98:2, 430-434

We give a description of the cubic polynomials f (x) with coefficients in the quadratic number fields $Q(\sqrt{-5})$ and $Q(\sqrt{15})$ for which the continued fraction expansion of the irrationality f (x) is periodic.

[2] V. P. Platonov, M.M.Petrunin, Y.Steinikov, V. S. Zhgoon, On the Finiteness of Hyperelliptic Fields with Special Properties and Periodic Expansion of $\sqrt{f}$

Doklady Mathematics, 2018, 98:3

We prove the finiteness of the set of square-free polynomials $f \in k[x]$ of odd degree distinct from 11 considered up to a natural equivalence relation for which the continued fraction expansion of the irrationality $\sqrt{f} \in k((x))$ is periodic and the corresponding hyperelliptic field $k(x)(\sqrt{f})$ contains an S-unit of degree 11. Moreover, it was proved for $k = Q$ that there are no polynomials of odd degree distinct from 9 and 11 satisfying the conditions mentioned above.

[3] F.Knop, V.S.Zhgoon, On complexity of reductive group actions over algebraically non-closed field and strong stability of the actions on flag varieties

Doklady Mathematics, 2018, to appear.

We annonce the results generalizing the Vinberg's Complexity Theorem for the action of reductive group on an algebraic variety over algebraically non-closed field. Also we give new results on the strong $k$-stability for the actions on flag varieties.

## 3.2 Scientific conferences and seminar talks

### 3.2.1 Alexander Belavin

[1] International Congress of Mathematicians- ICM-2018, in Rio de Janeiro, RJ, Brazil, 1–9 August,

Invited talk "Special geometry on Calabi-Yau moduli space and Q-invariant Milnor rings".

[2] International Workshop “Non-Perturbative Methods in Field Theory and String Theory.” Kyoto University, Kyoto, October 21 to October 27, 2018

Invited talk "Special geometry on the Moduli space of Calabi-Yau manifolds, Localization and Two-sphere partition functions".

### 3.2.2 Yurii Burman

No conferences.

### 3.2.3 Alexei Elagin

[1] School “Hodge theory Old and New”, Moscow, May 21 - June 7.

### 3.2.4 Konstantin Fedorovsky

[1] Workshop “Analysis Days in Piedmont”, Salmour, Italy, May 7–11, 2018

Talk (invited plenary talk) “Approximation of functions by solutions of second order elliptic systems on compact sets in the plane”

[2] Conference “International Conference Dedicated to 90-th Anniversary of Sergey Mergelyan”, Erevan, Armenia, May 20–25, 2018

Talk “Nevanlinna domains with large boundaries”

[3] International conference “Complex Analysis and Geometry”, Ufa, Russia, May 23–26, 2018

Talk (invited plenary talk) “Approximation of functions by solutions of elliptic systems”

[4] International Conference “Advanced courses in Operator Theory and Complex Analysis”, Bologna, Italy, June 11–14, 2018

Talk (invited talk) “Nevanlinna domains with large boundaries”

[5] International conference “27 St. Petersburg Summer Meeting in Mathematical Analysis”, Saint Petersburg, Russia, August 6–11, 2018

Talk (plenary talk) “$\mathrm{Lip}^m$-reflection of harmonic functions across boundaries of simple Caratheodory domains in $\mathbb{R}^n$”

[6] International conference “VII Russian–Armenian workshop on Mathematical Physics, Complex Analysis and Related Topics”

Talk (plenary talk) “Carathéodory domains, analytic balayage of measures and badly approximated functions in $L^p$”

[7] Talk “Carathéodory sets and analytic balayage of measures” at the seminar “Complex analysis in several variables” (Vitushkin Seminar), Moscow State University, April 11, 2018

[8] Talk "Three problems on univalent functions in model spaces" at "Seminari d'Analisi de Barcelona", Universitat de Barcelona & Universitat Autònoma de Barcelona, July 4, 2018

[9] Talk "Badly approximable functions in $L^p$ and analytic balayage of measures" at "Seminaire d'Analyse Fonctionnelle", l'Institut de Mathématiques de Jussieu–Paris Rive Gauche, November 15, 2018

[10] Talk "Univalent functions in model spaces" at "Seminaire d'Analyse Fonctionnelle", Université de Lille, November 16, 2018

[11] Talk "$\mathrm{Lip}^m$-reflection of harmonic functions over boundaries of simple Carathéodory domains in $\mathbb{R}^n$" at "Seminaire d'Analyse et Geometrie", Aix-Marseille Université, December 3, 2018

#### 3.2.5 Anton Fonarev

[1] Visit to University of Geneva, February

Talk "Derived categories of algebraic varieties for a working mathematician" on February 21. Talk "Embedding derived categories of curves into derived categories of moduli of stable vector bundles" on February 26.

[2] Visit to IPMU, University of Tokyo, November

Talk "On the generalised Dubrovin's conjecture for Grassmannians" on November 13.

[3] Seminar talk at the Shafarevich Semniar, Steklov Mathematical Institute

Talk "Generalized staircase complexes" on March 20.

#### 3.2.6 Alexander Kolesnikov

[1] Conference "Statistical optimal transport", Moscow, 24-25 July, 2018

Talk "Mass transportation: the classical Monge-Kantorovich problem and recent developments"

[2] Conference "Moscow - Pisa colloquium", Moscow, 1-5 October, 2018

Talk "Logarithmic Minkowski problem and optimal transportation"

[3] Conference," Laboratory of Stochastic Analysis Winter Meeting", Moscow, Snegiri, 3-7 December, 2018

Talk "On Minkowski-type problems"

#### 3.2.7 Mikhail Lashkevich

No conferences.

### 3.2.8 Taras Panov

[1] The 45th Symposium on Transformation Groups, Kumamoto, Japan, December, 6–8.

Plenary talk "Basic cohomology of canonical holomorphic foliations on complex moment-angle manifolds".

[2] IX International Conference of the Georgian Mathematical Union, Batumi and Tbilisi, Georgia, September, 3–7.

Plenary talk "Right-angled polytopes, hyperbolic manifolds and torus actions".

[3] Siberian Summer School "Current Developments in Geometry", Novosibirsk, Russia, August, 27–September, 1.

Invited lectures "Geometry and topology of toric varieties".

[4] International Conference/School "Glances at Manifolds 2018", Krakow, Poland, July, 2–6.

Series of invited lectures "Cobordims and group actions on manifolds".

[5] International Conference "Dynamics in Siberia-2018", Novosibirsk, Russia February, 26–March, 4.

[6] Visit to China, November.

Series of invited lectures "Cobordims and group actions on manifolds" (Fudan University, Shanghai).

Colloquim talk "Foliations arising from configuration of vectors, and topology of non-degenerate leaf spaces" (Fudan University, Shanghai).

### 3.2.9 Alexei Penskoi

[1] International conference "Algebraic Topology, Combinatorics, and Mathematical Physics" on occasion of Victor Buchstaber's 75th birthday, Moscow, May 2430, 2018

Talk "Isoperimetric inequalities for Laplace eigenvalues on the sphere and the real projective plane"

[2] Visit to Montréal, Québec, Canada, September 2018

Talk "Free boundary minimal surfaces and overdetermined boundary value problems" at Spectral Geometry Seminar at the Université de Montréal, September 20, 2018

[3] Geometry, Topology and Mathematical Physics Seminar, Moscow State University & Steklov Mathematical Institute of RAS.

Talk "Isoperimetric inequalities for Laplace eigenvalues on the sphere", February 7, 2018.

[4] Seminar of the Laboratory of Algebraic Geometry and its Applications, NRU HSE, Moscow

Talk "Isoperimetric inequalities for Laplace eigenvalues on the sphere", March 2, 2018.

[5] Krasilshchik Seminar "Geometry of Differential Equations", IUM

Talk "Isoperimetric inequalities for Laplace eigenvalues on the sphere", March 14, 2018.

[6] Sabitov Seminar “Global Geometry”, MSU

Talk “Isoperimetric inequalities for Laplace eigenvalues on the sphere”, March 16, 2018.

[7] Spectral Geometry Seminar, IUM

Talk “Free boundary minimal surfaces and overdetermined boundary value problems”, October 6, 2018

[8] Geometry and Topology Seminar, MSU

Talk “Free boundary minimal surfaces and overdetermined boundary value problems”, November 20, 2018

### 3.2.10 Vladimir Poberezhnyi

[1] Seminar “Analitical theory of differential equations” (Steklov Mathematical Institute)

Talk “On integrable hierarchies of pseudo difference operators”

### 3.2.11 Leonid Rybnikov

[1] Mini-course “Bethe ansatz and crystals” at Tokyo University May 21-2 2018 “Bethe ansatz and crystals”.

[2] Talk “Gaudin model and crystals”, at the Workshop: Vertex Algebras and Gauge Theory 2018-12-17 - 2018-12-21, Simons Center, Stony Brook University.

[3] Seminar talk “Gaudin model and crystals” at the Representation Theory Seminar, Kyoto University, June 18, 2018.

[4] Seminar talk “Shift of argument subalgebras and Kashiwara crystals” at the Lie Groups and Invariant Theory Seminar at Moscow State University, October 17, 2018

[5] Seminar talk “Bethe subalgebras and wonderful compactifications” at the“Riemannian manifolds, Lie algebras and Mathematical Physics” Seminar at the Independent University of Moscow November 23, 2018

### 3.2.12 George Shabat

[1] Institute of Philosophy, Russian Academy of Sciences, the round table ”Constructive knowledge 5: representation of knowkedge” , 14 2018.

The talk Geometric theorems: statements, proofs, comprehension (in russian).

[2] International algebraic conference in memory of A.G. Kurosh, May 25, 2018, MSU.

The talk Hurwitz existence problem and computer algebra.

[3] Invited seminar talk, 2018.6.14, Warsaw University, June 14, 2018.

Natural language in mathematics and physics.

[4] Vladimir Voevodsky Memorial Conference, September 11 - 14, 2018. Institute for Advanced Study, Princeton, USA.

The talk ”Galois, Grothendieck and Voevodsky”.

[5] IV International Conference "Current Problems of teaching mathematics and computer science in the schools and universities" , Moscow Pedagogical State University, December 3-5, 2018.

The plenary talk "Computer experiment in teaching mathematics" (in russian).

#### 3.2.13 Stanislav Shaposhnikov

[1] International Conference "Stochastic Partial Differential Equations"
(MarseilleLuminy, CIRM, 14.05.2018-18.05.2018)
Talk: "Nonlinear diffusion processes and Fokker-Planck-Kolmogorov equations"

[2] International Conference "6th Linnaeus University Workshop in Stochastic Analysis and Applications"
(Vaxjo, Linnaeus University, 06.06.2018-08.06.2018)
Talk: "Distances between transition probabilities of diffusios with partially degenerate diffusion matrices"

[3] International Conference "New Trends in Stochastic Analysis"
(Beijing, 11.09.2018-23.09.2018)
Talk: "Nonlinear Fokker-Planck-Kolmogorov equations"

#### 3.2.14 Mikhail Skopenkov

[1] Skopenkov M., "Conservation of energy in lattice field theories", Contemporary mathematics in honor of the 80th birthday of Vladimir Arnold (1937-2010), December 18-23, 2017, Moscow, Russia, http://me.hse.ru/lando/contmath2017/. (This conference did not appear in the previous-year report.)

[2] Skopenkov M., "Conservation of energy in lattice field theories", poster, Optimal control and differential games dedicated to the 110th anniversary of L.S. Pontryagin, December 12-14, 2018, Moscow, Russia, http://www.mathnet.ru/eng/conf1287.

[3] Talks on the same subject at several seminars in Moscow.

#### 3.2.15 Evgeni Smirnov

[1] School and conference "Lie algebras, algebraic groups and invariant theory", Samara, August 18-25, 2018

Talk: "Pipe dream complexes and slide polynomials"

[2] Lomonossov Moscow State University, Mechanics and Mathematics Department, seminar on Algebraic Topology and Applications (Postnikov seminar), October 16 and 23, 2018

Talk: “Subword complexes and slide polynomials”

#### 3.2.16 Ilya Vyugin

[1] Vyugin, I.V. ”On the Riemann Hilbert Problem for Difference and q-Difference” Systems Dynamics in Syberia 26.02.2018-4.03.2018 (http://www.math.nsc.ru/conference/ds/2018/Dynamics

[2] Vyugin, I.V., ”On the Polynomial Sum-Product Problem” (in Russian), Differential Equations and Related Topics, Kolomna, 15.06.2018-16.06.2018.

[3] Vyugin, I.V. ”On the Riemann-Hilbert Problem for Difference and q-Difference Systems”, International Conference on Differential Equations and Dynamical Systems, Suzdal, 6-11.07.2018 (http://agora.guru.ru/display.php?conf=diff-2018).

[4] Vyugin, I.V. ”On the Riemann-Hilbert Problem for Difference and q-Difference Systems”, XXXVII WORKSHOP ON GEOMETRIC METHODS IN PHYSICS BIAOWIEA, POLAND, 1-7.07.2018 (http://wgmp.uwb.edu.pl/).

#### 3.2.17 Vladimir Zhgoon

[1] Second memorial conference of A.Zykin (Moscow).

Talk: On the finiteness theorems for spherical varieties.

[2] Seminar uber Komplexe Geometrie (Bochum).

Talk: On complexity of algebraic varieties over algebraically non-closed fields

[3] Emmy-Noether-Seminars (Erlangen).

Talk: omplexity of algebraic varieties over algebraically non-closed fields and the action of the k-Weyl group on the set of orbits of minimal parabolic subgroup

[4] Seminar of Laboratory of Algebraic Geometry (Moscow)

Talk: On Compexity of homogeneous spaces over algebraically non-closed field.

## 3.3 Teaching

#### 3.3.1 Alexander Belavin

[1] Introduction to Quantum Field Theory. Independent University of Moscow, IV year students, September-December 2018, 2 hours per week.

Program

Lecture 1. Classical field theory. The idea of the field. Lagrangian. Action. Equations of motion. Symmetry. Theorem Noeter. The conservation laws. Poincare - invariance. Energy-Momentum Tensor. Lorenz group generators. Hamiltonian formalism.

Lecture 2. Quantum theory of Klein-Gordon field. Canonical quantization. Creation-destruction operators of particles. Operators H and P ?. Commutation relations. Fock space and energy spectrum. The idea of ??circumcision. The energy of zero-point oscillations and the Casimir effect.

Lecture 3. Green functions. Heisenberg field operators. Feynman propagator. Its analytic properties in the complex time plane. Causality.

Lecture 4. QFT in Euclidean space. The connection of the Green functions of QFT in Minkowski space and in Euclidean space. Feynman path Integral in Quantum mechanics.

Lecture 5. Functional integral in QFT. The connection between the QFT and the classical Statistical physics. Representation for the Feynman propagator in the form of an integral over paths. Calculation of functional integrals.

Lecture 6. Functional integral in the theory of Klein-Gordon. The Wick theorem and Feynman diagrams. The perturbation theory. The theory $\lambda\phi^4$. Diagramme technique.

Lecture 7. Feynman rules. Perturbation theory. Connected and unconnected diagrams.

Lecture 8. The examples and classification of Feynman diagrams.

Lecture 9. The Dirac theory. Quatization of the Dirac theory. Dirac propagator.

Lecture 10. Feynman rules for fermions.

Lecture 11. The Yukawa theory. Yukawa potential. Feynman rules in Quatum Electrodynamic. The Coulomb potential.

Lecture 12. Calculations of the contributions of diagrams. Momentum presentation. The idea of the regularization of the Feymann integrals. The idea of the renormalization. Counterterms.

Lecture 13. Renormalization program. Types of divergences. Renormalizable and non-renormalizable field theories. Methods of the regularization.

Lecture 14. One-loop order approximation. Dimensional regularization. Renormalization schemes. Normalization conditions. 2- and 4-point Green functions in the one-loop approximation.

#### 3.3.2 Yurii Burman

[1] Topology. Independent University of Moscow, I year students, September-December 2018, 2 hours per week.

Program

1. What is topology for?

   Examples of topological statements: Brouwer theorem in dimensions 1 and 2, fundamental theorem of algebra, hairy ball theorem.

2. What is continuity.

   Topological spaces and continuous mappings. Homeomorphism. Connected spaces, connectedness of a segment, arcwise connected spaces.

3. Topological spaces.

   Metric spaces. Induced topology, quotient topology, direct product. Gluing of topological spaces, join and the like.

4. Homotopies, homeomorphisms and homotopy equivalence.

   Category of topological spaces. Homotopy category. Homotopy invariance of connectedness and arcwise connectedness.

5. Compactness.

   The main properties of compact spaces. Mapping from and to compacts, the "closed/compact" duality.

6. Fundamental group.

   The fundamental groupoid and the fundamental group; their behavior under maps. $\pi_1(S^1)$. The Van Kampen theorem.

7. Coverings.

   The category of coverings. The covering homotopy theorem. The correspondence between coverings over a given base and subgroups of the fundamental group of the base. The knot group.

8. Higher homotopy groups.

   The group $\pi_k(S^n)$ for all $k \leq n$. The degree of a map. The linking number. The group $\pi_3(S^2)$.

### 3.3.3 Alexei Elagin

[1] Algebra – 2. Independent University of Moscow, 1st year students, Spring 2018. Program:

1. **Linear algebra.** Vector spaces, free modules. Dual vector space. Dual linear map and dual basis. Non-degenerate pairings. Operators. Eigenvectors and eigenvalues. Semi-simple operators. Characteristic polynomials. Cayley–Hamilton theorem. Root subspaces. Jordan normal form. Bilinear and quadratic forms. Symmetric and skew-symmetric bilinear forms. Quadratic forms over $\mathbb{R}$ and $\mathbb{C}$. Operators in euclidean spaces. Adjoint operator. Orthogonal and self-adjoint operators. Quadratic forms in euclidean spaces. Principal axis. Tensor multiplication of modules. Operators and bilinear form as tensors. Graded algebras, ideals and modules. Tensor algebra of a vector space. Symmetric and exterior algebra. Tensor, symmetric and exterior power of a vector space.

2. **Representation theory.** Linear repesentations of groups. Irreducible representations. Operations with representations. Schur lemma. Maschke theorem. Representations of some groups. Morphisms of representations. Regular representations. Burnside formula. Characters of representations, orthogonality relations. Number of irreducible representations. Algebras over a field. Group algebras. Quaternion algebra, division algebras, Frobenius theorem. Simple and semi-simple modules. Semi-simple algebras. Simple algebras. Criteria for semi-simplicity. Trace form.

3. **Field theory.** Field extensions. Algebraic elements, their degree. Finite and algebraic field extensions. Degree of an extension. Tower of extensions. Minimal polynomial of an element. Splitting field of a polynomials.

See *http://ium.mccme.ru/s18/s18-algebra2.html* for the exercise sheets.
[2] Algebra – 3. Independent University of Moscow, 2nd year students, Fall 2018.
Program:

1. **Field Theory.** Algebraic closure of a field. The fundamental Theorem of algebra. Separable field extensions. Separable degree. Embeddings and automorphisms of fields. Normal extensions. Galois extensions. Galois group. Primitive elements. Galois theory. Solving polynomial equations in radicals. Finite fields. Frobenius automorphism.

2. **Algebraic geometry.** Affine space. Affine algebraic sets. Irreducible algebraic sets. Ideals of algebraic sets. Prime ideals. Krull dimension of algebraic sets. Regular functions. Regular maps. Nullstellensatz. Maximal ideals in polynomial algebras. Hilberts's basis theorem. Noetherian rings. Finite ring extensions. Integral elements. Finite maps of algebraic sets.

3. **Homological algebra.** Categories, functors. Natural maps of functors. Equivalence of categories. Representable functors. Adjoint functors. Projective and injective modules. Exact triples. Exact, right exact, left exact functors on module categories. Complexes, maps of complexes. Cohomology. Long exact sequence of cohomology. Homotopic maps of complexes. Acyclic complexes. Resolutions. Right and left derived functors. Functors Ext and Tor. Long exact sequences of Ext and Tor. Kozhul complex. Global dimension of a ring. Hilbert's syzygy theorem.

See *http://ium.mccme.ru/f18/f18-algebra3.html* for the exercise sheets.

#### 3.3.4 Konstantin Fedorovsky

[1] Complex Analysis. Independent University of Moscow, II year students, February–May 2018, 2 hours per week.

Program:

1)Complex number, their properties and operations on them. Complex plane $\mathbb{C}$ and its compactification $\overline{\mathbb{C}}$. The function $e^z$ and exponential form of complex numbers. Paths and curves in $\mathbb{C}$. Increment of the argument along a path. Index of a path and its properties.

2) Differentiability of functions of a complex variable. Cauchy–Riemann conditions. Properties of a complex derivative. Holomorphic functions. Conformity and its relationship with holomorphy. Basic elementary functions of a complex variable and thier properties. Multi-valued functions and their continuous and holomorphic branches.

3) Integral over a path and over a curve by a complex variable and their properties. Goursat lemma. Cauchy integral theorem. Complex primitive, its properties and Newton–Leibnitz formula. Existence of holomorphic branches of the root function and of the logarithm in simply connected domains in $\mathbb{C} \setminus \{0\}$.

Cauchy integral formula, Cauchy formula for derivatives and infinite differentiability of holomorphic functions. Pompeiu formula. Mean value theorem, maximum modulus principle. Morera theorem. Local uniformly convergence of sequences of holomorphic functions. Weierstrass theorem.

4) Power series, Cauchy–Hadamard formula. Termwise differentiability and integrability of power series. Singular points at the boundary of the disk of convergence. Pringsheim theorem.

Taylor series. Expansion of holomorphic function into a power series. Cauchy inequalities for Taylor coefficients. Liouville theorem. Zeros of holomorphic functions. Uniqueness theorem. Approximation of holomorphic functions by polynomials, Runge's theorem.

5) Laurent series. Expansion of holomorphic functions into a Laurent series. Cauchy inequalities for Laurent coefficients. Isolated singularities of holomorphic functions and their classification, Sokhotskii theorem. Infinity as a singular point, entire and meromorphic

functions with poles at infinity. Schwarz lemma and conformal automorphisms of the basic domains.

Residues. Cauchy residues theorem. Residue at $\infty$. Evaluation of residues. Jordan lemma. Evaluation of integrals (including integrals in the sense of principal values) using the method of residues. Logarithmic residue. Argument principle. Rouche theorem. Domain preservation principle.

6) Univalent functions and their basic properties (criteria for local and global univalence, Hurwitz theorem and its corollaries, area theorem and Koebe theorem).

7) Construction of conformal mappings of a given domains. Inverse boundary correspondence principle for conformal mapping. Riemann–Schwarz symmetry principle. Montel theorem. Riemann mapping theorem. Boundary behavior of conformal mappings. Carathéodory theorems.

8) Analytic elements and their analytic continuation. Analytic continuation along a path and by a chain. Theorem about continuation by homotopic paths and monodromy theorem.

Complete analytic function in the sense of Weierstrass, its holomorphic branches. Branch points of analytic functions and their classification. Complete analytic function 'root' and 'logarithm'. The concept of a Riemann surface.

Addendum: Modular function, Picard theorems. Harmonic functions and their properties. Entire functions: growing of entire function, order and type of entire function; decomposition into infinite product.

[2] Theory of functions of a complex variable, Bauman Moscow State Technical University, II year students, February–June 2018, 3 hours per week.

1) Complex numbers, complex plane. Elements of topology of the complex plane. Increment of the argument along a path. Index of a path with respect to a point and its properties.

2) Differentiability of functions of a complex variable. Cauchy–Riemann conditions. Holomorphic functions. The notion of a conformal mapping. Elementary functions of a complex variable and their properties.

3) Integration of complex functions over a curve. Complex primitive. Goursat lemma. Cauchy integral theorem. Cauchy integral formula. Mean value theorem. Cauchy formula for derivatives and infinite differentiability of holomorphic functions. Morera theorem.

4) Power series and their domains of convergence. Holomorphicity of the sum of a power series. Taylor series expansion of a holomorphic function. Cauchy inequalities. Liouville theorem.

5) Loran series and their domains of convergence. Cauchy inequalities for coefficients of Loran series. Isolated singularities of holomorphic functions and their description. Sokhotskii theorem. Infinity as a singular point.

6) Residues (definition, basic properties and formulae for evaluation of residues). Cauchy residue theorem. Residue at the infinity and theorem on the total sum of residues. Residue

with respect to a domain. Jordan lemma. Evaluation of integrals using the method of residues.

7) Logarithmic residue and its properties. Argument principle. Rouché theorem. Maximum modulus principle and domain preservation principle.

8) Criteria of univalence and local invertibility. Hurwitz theorem and its corollaries. Properties of univalent functions.

9) The concept of a conformal mapping. Elementary functions and respective conformal mappings. Inverse boundary correspondence principle for conformal mappings. Riemann-Schwarz symmetry principle and its applications.

10) Schwarz's lemma and evaluation of groups of conformal automorphisms of basic domains. Riemann theorem and Carathéodory extension theorem.

11) The concept of analytic continuation. Weierstrass theory. Analytic functions and their singularities.

12) Laplace transform, and its properties. Using the operational calculus for solving ordinary differential equations.

#### 3.3.5 Anton Fonarev

[1] Basic Algebraic Geometry, Math in Moscow, Spring 2018.
- Zariski topology, affine algebraic varieties.
- Hilbert's basis theorem, Noëther normalization, Hilbert's Nullstellensatz.
- Projective varieties.
- Categories, functors, natural transformations.
- Sheaves, glueing algebraic varieties.
- Rational functions.
- Line bundles, Cartier divisors, Weil divisors.
- Morphisms, morphisms into $\mathbb{P}(V)$.
- Local properties of varieties, tangent spaces.
- Vector bundles, locally free sheaves, coherent sheaves.
- Čech cohomology.
- Degree, coherent sheaves on projective varieties.
- Riemann–Roch for curves.
- Rough classification of curves.
- Blow-up, resolution of singularities, examples of surfaces.

[2] Basic Algebraic Geometry, Math in Moscow, Fall 2018.
- Zariski topology, affine algebraic varieties.
- Hilbert's basis theorem, Noëther normalization, Hilbert's Nullstellensatz.
- Projective varieties.
- Categories, functors, natural transformations.
- Sheaves, glueing algebraic varieties.
- Rational functions.

• Line bundles, Cartier divisors, Weil divisors.
• Morphisms, morphisms into $\mathbb{P}(V)$.
• Local properties of varieties, tangent spaces.
• Vector bundles, locally free sheaves, coherent sheaves.
• Čech cohomology.
• Degree, coherent sheaves on projective varieties.
• Riemann–Roch for curves.
• Rough classification of curves.
• Blow-up, resolution of singularities, examples of surfaces.

### 3.3.6 Alexander Kolesnikov

[1] Sobolev spaces, convex analysis, and probability. Independent University of Moscow, III-IV year students, September-December 2018, 2 hours per week.

In this course we present geometric constructions and corresponding analytical instruments appearing in probability theory and stochastic processes. We show how the classical problems and theories (isoperimetric problem, Minkowski theory, Sobolev spaces) interact with modern ideas (optimal transportation, transportation inequalities, calculus on manifolds, metric-measure spaces, Gamma-2 calculus). We discuss a broad spectrum of various instruments from geometric analysis (semigroups, mass transportation, various functional inequalities etc.), applications, and famous open problems from convex analysis, motivating development of this field.

1. Euclidean isoperimetric inequality. Brunn–Minkowski inequality.

2. Mass transportation and Kantorovich problem. Monge–Ampère equation.

3. Sobolev spaces in Euclidean space. Classical Sobolev inequalities.

4. Semigrous. Model spaces (Euclidean space, sphere, hyperbolic space, Ornstein–Uhlenbeck semigroup).

5. Gaussian measures. Sudakov–Tsirelson inequality. Sobolev spaces with respect to Gaussian measures. Gaussian concentration.

6. Log-concave measures. Prekopa–Leindler theorem.

7. Manifolds with measures. Tensor Ricci and curvature-dimension condition. Bochner formula, Gamma-2 calculus.

8. Poincaré inequality. Examples and basic properties. Brascamp–Lieb inequality.

9. Logarithmic Sobolev inequality. Sobolev inequalities on manifolds.

10. Transport inequalities.

11. Isoperimetric inequalities and concentration of measures.

12. Applications. Open problems of convex analysis (KLS, hyperplane conjecture, log-Brunn–Minkowski inequality).

**Bibliography**

[1] Bakry D., Gentil I., Ledoux M., Analysis and geometry of Markov diffusion operators. Springer-Verlag. Grundlehren der Mathematischen Wissenschaften. Vol. 348, 2014.

[2] Adams R.A, Fournier J.J., Sobolev spaces. AMS. 2003.
[3] Bogachev V.I., Gaussian measures. AMS 1998.
[4] Villani C., Topics in optimal transportation. AMS. Graduate Studies in Mathematics. Vol. 58, 2003.
[5] Artstein-Avidan S., Giannopoulos A., Milman V.D., Asymptotic geometric analysis, Part I, Mathematical Surveys and Monographs, Vol. 202, AMS, 2015.
[6] Ledoux M., Concentration of measure phenomenon. AMS. Matematical surveys and Monographs , Vol. 89, 2001.
[7] Brazitikos S., Giannopoulos A., Valletas P., Vrisiou B.-H., Geometry of Isotropic Convex Bodies. AMS. Mathematical Surveys and Monographs, Vol. 196, 2014.

### 3.3.7 Mikhail Lashkevich

[1] General Relativity Theory. III year students, September–December 2018, 3 hours per week.

#### Program

The texts of the lecture can be found at:
`https://homepages.itp.ac.ru/l̃ashkevi/lectures/gr18/`.

1. Geometry and physics of special relativity theory:
   - nondegenerate symmetric form in a linear space and signature;
   - metric in an affine space, index and indexless notation;
   - principle of least action for particles and fields.

   **Seminar:** Lorentz transformation, coordinate systems, metric.

2. Main concepts of differential geometry and space-time
   - manifolds, tangent bundles, tensor fields;
   - affine connection and covariant derivatives;
   - metric and signature;
   - Levi-Civita connection.

   **Seminar:** Physical interpretation of metric: time and space intervals, synchronization of clocks.

3. Riemann curvature. Transformations of tensor fields:
   - Riemann curvature tensor, Ricci tensor and Ricci scalar;

- properties of the curvature tensor, Bianchi identities;
- transformation of coordinates and Lie derivative.

**Seminar:** Symmetries of metric and Killing vector fields.

4. Particles in a curved space-time:
    - a particle in a curved space-time, geodesics, external electromagnetic field;
    - Hamilton–Jacobi equation.

    **Seminar:** Hamilton–Jacobi equation, separation of variables and integrability.

5. Fields in a curved space-time and energy-momentum tensor:
    - Action and equations of motion;
    - Canonical energy-momentum tensor in flat space-time;
    - Energy-momentum tensor in curved space-time and its covariant conservation.

    **Seminar:** Energy-momentum tensor for different physical systems

6. Equations of gravitation field and conservation laws:
    - Hilbert–Einstein action;
    - Einstein equations, their structure, number of degrees of freedom;
    - Energy-momentum pseudotensor and conservation laws.

    **Seminar:** Total energy of stationary system.

7. Weak gravitational field:
    - linearized Einstein equations, gauge fixing;
    - static solutions to linearized Einstein equations, residual gauge freedom;
    - energy and angular momentum via asymptotics of gravitational field.

    **Seminar:** Problems on gravitational field in the linear approximation.

8. Gravitational waves:
    - free solutions of homogeneous linearized Einstein equations;
    - plane waves, monochromatic waves, gauge fixing, polarizations;
    - energy-momentum pseudotensor of a plane gravitational wave.

    **Seminar:** Strong gravitational wave.

9. Emission of gravitation waves:
   - retarded solution to the wave equation, simplification in dimension 4;
   - non-relativistic source: restrictions by the energy-momentum conservation and quadrupole emission;
   - angular distribution and calculation of the total emitted energy.

   **Seminar:** Interaction of gravitational waves with condensed matter and electromagnetic field.

10. Schwartzschild solution:
    - spherically symmetric Einstein equation, its direct solution;
    - geodesics in the Schwartzschild metric, incompleteness of the Schwartzschild coordinates;
    - Eddington–Finkelstein coordinates, event horizon and singularity;
    - Kruskal coordinates and Penrose diagram, maximally extended Kruskal manifold.

    **Seminar:** Gravitational field of static spherically symmetric body, static equilibrium condition.

11. Motion of a particle in the Schwartzschild metric:
    - solution of the Hamilton–Jacobi equation;
    - four types of motion in the Schwartzschild metric, conditions for their realization.

    **Seminar:** Fall of a layer of dustlike matter on a black hole.

12. Motion in a rather weak gravitational field and experimental checks of general relativity:
    - nearly-Newtonian field and perihelion precession;
    - deviation of a light beam in a weak gravitational field.

    **Seminar:** Isotropic hypersurfaces: invariance of images.

13. Charged and rotating black holes:
    - Reissner–Norstrm solution of Maxwell and Einstein equations;
    - singularity, event horizon, Cauchy horizon, Penrose diagram;
    - Kerr–Newman solution, ergosphere, ring singularity, Penrose diagram.

**Seminar:** Isotropic hypersurfaces: evolution of images.

14. Cosmological solutions. Friedmann models:

    - homogeneous and isotropic Universe, constant curvature spaces;
    - Friedmann equations, their solutions for dustlike matter and ultrarelativistic gas;
    - Cosmological constant and accelerating expansion, modern model of expanding Universe, dark matter and dark energy.

    **Seminar:** Difficulties of the Friedmann models. Inflation theory.

### 3.3.8 Taras Panov

[1] Topology-1. Independent Independent University of Moscow, I year students, February–May 2015, 2 hours per week.

Program:
1. Necessary facts from point-set topology.
2. Operations on topological spaces.
3. Homotopy and homotopy equivalence.
4. Cellular (CW) complexes.
5. Fundamental group.
6. Van Kampen Theorem.
7. Fundamental group of cellular complexes.
8. Coverings.
9. Fibrations.
10. Homotopy groups.

http://higeom.math.msu.su/people/taras/teaching/panov-topology1.pdf

[2] Topology-2. Independent Independent University of Moscow, II year students, September–December 2015, 2 hours per week.

Program:
1. Simplicial homology.
2. Singular homology.
3. Cellular homology.
4. Homotopy groups and homology groups.
5. Cohomology and multiplications.

http://higeom.math.msu.su/people/taras/teaching/panov-topology2.pdf

[3] Algebraic topology. Part II (advanced course), Department of Mathematics and Mechanics, Moscow State University, II–VI year students, February–May 2018, 2 hours per week

Program: same as [2] Topology-2.

[4] Complex cobordism and torus actions. Part I (advanced course), Department of Mathematics and Mechanics, Moscow State University, II–VI year students, September–December 2018, 2 hours per week

Program:

1. Bordism of manifolds.
2. Thom spaces and cobordism functors.
3. Oriented and complex bordism.
4. Characteristic classes and numbers.
5. Cohomology operations.
6. Structure results.

### 3.3.9 Alexei Penskoi

[1] Complex analytic manifolds and holomorphic vector bundles-II, Independent University of Moscow, 3-6 year students, February-May 2018, 2 hours per week.

Program.

1. Holomorphic linear bundles and divisors.
2. Harmonic theory on compact manifolds.
3. Hodge decomposition on compact Kähler manifolds.
4. Hodge Riemann bilinear relations on compact Kähler manifolds.
5. Hodge manifolds.
6. Kodaira theorem.

[2] Advanced geometry, Independent University of Moscow, 3-6 year students, September-December 2018, 2 hours per week.

Program.

1. Vector bundles and Chern-Weil construction of characteristic classes.
2. Elements of $K$-theory (Grotendieck group of an abelian monoid, $K$-group of a manifold, $K_c$-group, Chern character with compact support.

3. Differential operators on sections of vector bundles. Symbol of a differential operator. Analytic and topological index. Atiyah-Singer theorem (without proof). Example: $d + d^*$ and Gauß-Bonnet theorem.

4. Clifford algebra, spinors and spinor structure, Dirac operator.

[3] Differential Geometry. Math in Moscow program of the Independent University of Moscow for undergraduate students from the U.S. and Canada, February-May 2018, 4 hours per week (lecture 2 hours + exersise class 2 hours).

1. Plane and space curves. Curvature, torsion, Frenet frame.

2. Surfaces in 3-space. Metrics and the second quadratic form. Curvature.

3. Connections in tangent and normal bundles to a k-surfaces in $\mathbf{R}^n$.

4. Parallel translations.

5. Geodesics.

6. Gauß and Codazzi formulas. "Theorema egregium" of Gauß.

7. Gauß-Bonnet theorem.

8. Extremal properties of geodesics. Minimal surfaces.

9. Levi-Civita connection.

10. Exponential map.

[4] Calculus on manifolds. "Math in Moscow" program at the Independent University of Moscow for undergraduate students from the U.S. and Canada, September-December 2018, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program

1. Definition and examples of smooth manifolds.

2. Orientability and orientation.

3. Tangent vectors and tangent space to a manifold at a point. Tangent bundles. Vector fields.

4. Skew-symmetric forms on linear spaces. Wedge product.

5. Differential forms on manifolds. Exterior differential.

6. Smooth maps of manifolds. Diffeomorphisms. The transformation rule under coordinate change for functions, vector fields and differential forms.

7. Integration. Coordinate change in the integral. Integration of differential forms. Stokes theorem. Green's formula, Gauss-Ostrogradskii divergence theorem, Stokes formula for a surface in $\mathbb{R}^3$.

8. Closed and exact forms. The Poincare lemma. De Rham cohomology.

[5] Minimal surfaces, Moscow State University, September-December 2018, 2 hours per week

1. Basics of differential geometry of surfaces.

2. Area functional and its variation. Minimal surfaces.

3. Conformal coordinates. Minimal surfaces in conformal coordinates.

4. Hopf quadratic differential.

5. Analytic propertis of minimal surfaces.

6. Weierstrass representations of minimal surfaces.

7. Branch points, their order.

8. Boundary problems, Plateau and free boundary problems.

[6] Exercise classes for various courses at National Research University — Higher School of Economics: Topology-I, January-March 2018, 2 hours per week. Topology-I, September-December 2018, 3 hours per week.

#### 3.3.10 Vladimir Poberezhnyi

[1] Calculus. Independent University of Moscow, I year students, September-December 2018, 2 hours per week.

Program

1. Metric spaces

2. Topological spaces

3. Limits and accumulation points

4. Real numbers

5. Completeness and completion
6. Compact spaces
7. Continuous functions on an interval
8. Differentiation and derivatives
9. Properties of differentiable maps
10. Around finite difference theorem
11. Higher order derivatives
12. Implicit function theorem

[2] With I.V.Vyugin Differential equations and isomonodromic deformations, Higher school of economics, III-VI year students, September-December 2018, 2 hours per week.

Program.

1. Complex differential equations
2. Regular and Fuchsian equations
3. Monodromy
4. Levelt theory
5. Vector bundles
6. Meromorphic connections
7. Riemann-Hilbert problem

#### 3.3.11 Leonid Rybnikov

[1] Representations of Lie Algebras (joint with Boris Feigin). Independent University of Moscow (joint with PhysTech), 3-4 year students (mostly from PhysTech), February-May and September-December 2018, 2 hours per week.

**Program.**

1. Transformation groups, examples.
2. Homogeneous spaces, examples.
3. Lobachevsky space and its isometry group.
4. Riemannian homgeneous spaces and maximal compact subgroups of Lie groups.
5. Classical compact groups.
6. Maximal tori and root systems.
7. Exceptional Lie group $G_2$.
8. Finite dimensional representations of compact groups.
9. Category $O$.
10. Explicit constructions of fundamental representations for classical Lie algebras.
11. Automorphisms of simple Lie algebras. Representation theory of $G_2$.

#### 3.3.12 George Shabat

[1] Cohomology of algebraic manifolds. Independent University of Moscow, 3-5 year students, February-May 2018, 2 hours per week.

Program

0. General overview
1. The necessary algebro-geometric concepts
2. The elements of combinatorial topology
3. Axioms of the (co)homology theories
4. Cohomology of the smooth and complex manifolds
5. Cech cohomology
6. Sheaf cohomology
7. Algebraic coherent sheaves
9. Cohomology of curves and surfaces
10. Structures in the cohomology of algebraic manifolds

[2] Algebra-1. Independent University of Moscow, 1 year students, September-December 2018, 2 hours per week.

Program

0. Introduction: equations, classification
1. Categorical language in algebra

2. Abstract groups
3. Groups acting on sets
4. Group extensions
5. Silow theory and classification of groups of order ¡ 60
6. Rings
7. Ideals
8. Finitely generated algebras
9. Systems of polynomial equations. Functor of points

#### 3.3.13 Stanislav Shaposhnikov

1) Mathematical calculus. Independent University of Moscow, 2 year students, September – December 2018, 4 hours per week.

Program:

1. Inverse function theorem. Implicit function theorem.
2. Smooth surfaces. Tangent space.
3. Topological spaces. Manifold.
4. Smooth functions. Whitney embedding theorem.
5. Sard theorem. The degree of a smooth mapping.
6. The area formula and the coarea formula.
7. Integrating by parts formula. Sobolev derivatives.
8. Smooth vector fields. Frobenius theorem. Lie theorem.
9. Differential forms. Stokes theorem and applications.

#### 3.3.14 Mikhail Skopenkov

[1] Introduction to graph theory. Independent University Moscow, I-III year students, Spring 2018, 2 hours per week. The course is supported by the fellowship.

Program (short version). Basic definitions. Counting in graphs. Paths in graphs. Counting trees. Isomorphism. Planar graphs. Graphs on surfaces. Coloring of graphs. Chromatic number, index, polynomial.

[2] Introduction to discrete mathematics. Higher School of Economics, I year students, Spring 2018, 4 hours per week.

Program (short version).

1. Intro to combinatorics: counting, the Pascal triangle, linear recurrences, generating functions.

2. Intro to probability: classical probability, independence, random variables, the Bernoulli process, law of large numbers.

3. Intro to graphs: number of edges, paths and cycles, connected graphs, trees.

[3] Elementary lattice field theory (aka Quarks game). Higher School of Economics, I-III year students, Spring 2018, 2 hours per week.

This is a play-based introduction to basic ideas of field theory describing elementary particles. It is going to give better intuition in both physics and mathematical branches such as differential geometry and complex analysis. For each particular theory and each new notion, we try to show how they appear naturally in a solution of a practical problem and which are their further applications. This makes most of the objects visual and simple.

The material is studied via problem solving, with detailed hints and discussion in classes. No prerequisites in physics are assumed; knowledge of school-level mathematics is sufficient. The course is accessible for 1st year students.

Program.

1. Toy model of lattice gauge theory: exchange of goods between cities. Relation to magnetic field. Quantization: random exchange rates. Exact solution of 1- and 2-dimensional lattice gauge theory. Numerical analysis in dimension 3 and 4. An example of non-Abelian gauge theory. Confinement of quarks. The essence of the problem in Yang-Mills theory (one of the "Millenium problems"). Higgs mechanism*. Strong- and weak-coupling expansions*.

2. Mathematical model of an electrical network - the simplest lattice field theory. Existence and uniqueness of the potential in an electrical network. Maximum principle. Energy conservation. Variational principle. Magnetic field. Relation to toy gauge theory. Discrete harmonic and discrete analytic functions.

[4] Elementary lattice field theory (aka Quarks game). Higher School of Economics, I-III year students, Fall 2018, 2 hours per week. Repetition of [3], point 1.

In addition to [1]–[4]: assistance teaching in basic courses at Higher School of Economics (I-III year students, approximately 7 hours per week throughout 2018), supervision of 10 undergraduate students.

### 3.3.15 Evgeni Smirnov

[1] Linear algebra, 1st year, 1st semester, Higher School of Economics, Department of Physics, September–December 2018, 2 hours of lectures and 2 hours of exercise sessions per week

Course outline:

1. Systems of linear equations. Gaussian elimination, row-echelon form.
2. Vector spaces, subspaces. Linear dependence and independence.
3. Linear maps, matrices, multiplication of matrices
4. Determinants.
5. Eigenvectors, eigenvalues, diagonalization of linear operators.
6. Bilinear forms, Jacobi's theorem, Sylvester's criterion.

7. Euclidean and Hermitian spaces.

8. Tensors, tensor product of vector spaces.

[2] Symmetric Functions, Higher School of Economics, course for 2nd–4th year students, September–December 2018, 4 hours per week

Course outline:

1. Symmetric polynomials. Elementary and complete symmetric polynomials, duality. Skew-symmetric polynomials. Schur polynomials.

2. Pieri, Giambelli, Jacobi–Trudi formulas.

3. Combinatorial description of Schur polynomials. Standard and semistandard Young tableaux. Kostka numbers. Hook length formula.

4. Danilov–Koshevoy massifs. Robinson–Schensted–Knuth correspondence via massifs. Littlewood–Richardson rule.

5. Schubert polynomials. Positivity. Pipe dreams, Fomin–Kirillov theorem, Monk's rule. Pipe dream complexes.

[3] Algebra, 1st year, 2nd semester, Higher School of Ecomonics, January–June 2018, 3 hours of lectures and 3 hours of exercise sessions per week

Course outline:

1. Finite Abelian groups

2. Eigenvectors and eigenvalies

3. Jordan and Frobenius normal forms

4. Classification of finitely generated modules over Euclidean rings

5. Bilinear and quadratic forms over R and C. Symmetric/Hermitian, orthogonal/unitary operators

6. Tensors. Tensor algebra. Symmetric and exterior algebras.

[4] Lie Groups, Independent University of Moscow, course for 2th–4th year students, September–December 2018

Course outline:

1. Lie groups. Definitions. Lie group actions on manifolds, orbits, homogeneous spaces. Classical groups.

2. Lie algebras. Tangent spaces to Lie groups. The exponential map.

3. Representations of Lie groups and Lie algebras. Representations of compact groups, complete reducibility.

4. Universal enveloping algebra. PBW theorem. Solvable and nilpotent Lie groups. Lie and Engel's theorems.

5. Complex semisimple Lie groups. Complete reducibility of their representations. Root decomposition.

6. Root systems. Classification of root systems, Serre's relations, classification of semisimple Lie algebras.

### 3.3.16 Ilya Vyugin

[1] Differential Geometry, Independent University of Moscow, Spring 2018.

Program

1. Geometry of sumanifolds.
2. Connections in vector bundles.
3. Riemann geometry.
4. Symplectic geometry.

[2] Smooth Manifolds (lectures). Higher School of Economics, second year students, Autumn 2018, 3 hours per week.

Program

0. Curves.
1. Manifolds. Tangent vectors.
2. Submanifolds.
3. Differential forms on manifolds.
4. Commutators. Frobenius theorem.
5. Integral of the differential form. Stokes theorem.
6. De Rham cohomology. Poincaré theorem.

[3] Complex Differential Equations and isomonodromy deformations (joint with V. Poberezhny) Special course in HSE, Spring 2018.

Program

1. Introduction to the analityc theory of differential equations.
2. Normal forms of lenear differential and difference equations.
3. Riemann-Hilbert problem and isomonodromy deformations..

### 3.3.17 Vladimir Zhgoon

[1] Advanced Algebra. Independent University of Moscow, Math in Moscow, Spring 2018, 2 hours per week.

Program

1. Basic group theory. Cosets, quotients, normal subgroups.
2. Existence of the elements of prime order.
3. Actions of the finite groups on sets. Orbits, stabilizers.
4. Actions of p-groups on finite sets and the number of fixed points.
5. Sylow's theorems. Existence, conjugacy, number of Sylow subgroups.
6. Simple groups. Solvable groups. Nilpotent groups.

7. Algebraic extensions of fields.
8. Separable field extensions. Normal extensions.
9. Galois extensions. Galois correspondence.
10. Solvability of roots of the polynomials in radicals.
11. Complements to the course on basic representation theory.
12. Irreducible representations. Schur's lemma.
13. Semisimple algebras and modules.
14. Group algebra. The Maschke theorem.
15. Application of the theory of characters to the structure of finite groups. Solvability of the group of order pnqm

[2] Spherical varieties. Higher school of economics, Spring 2018, 2 hours per week.

- Scherical varieties. General properties and equivalent definitions.
- Local structure of spherical varieties.
- Akhiezer theorem on modality and complexity. Vinberg theorem on complexity. Horospherical contraction.
- Finiteness of number of B-orbits in spherical variety. Springer-Richardson monoid. Action of the Weyl group on the set of $B$-orbits. BottSamelson resolutions of Schubert varieties.
- Luna-Vust theory of spherical embeddings. Invariant valuations, colours.
- Demazure construction of wonderful compactifications. Bialynicki-Birula cell decomposition.
- Equivariant geometry of cotangent bundle. Structure of the moment map. Variety of generic horosheres and the little Weyl group.
- Moment polytope and Brion's polytope.

[3] Advanced Algebra. Independent University of Moscow, Math in Moscow, September-December 2018, 2 hours per week.

Program
1. Basic group theory. Cosets, quotients, normal subgroups.
2. Existence of the elements of prime order.
3. Actions of the finite groups on sets. Orbits, stabilizers.
4. Actions of p-groups on finite sets and the number of fixed points.
5. Sylow's theorems. Existence, conjugacy, number of Sylow subgroups.

6. Simple groups. Solvable groups. Nilpotent groups.
7. Algebraic extensions of fields.
8. Separable field extensions. Normal extensions.
9. Galois extensions. Galois correspondence.
10. Solvability of roots of the polynomials in radicals.
11. Tensor products. Symmetric and alternating powers.

[4] Algebra and Arithmetics. Higher school of economics, September-December 2018, 2 hours per week.

(1) Basic notions of integer numbers and residues
(2) Chinese remainder theorem, little Ferma's theorem, Wilson's lemma.
(3) Quadratic residues. Gauss reciprocity law.
(4) Basic notions of group theory. Cosets, normal, nilpotent and solvable subgroups.
(5) Group actions. Orbits, stabilizers, normalizers, conjugacy classes. Burside formula.
(6)* Sylow theorems
(7) Basic notions of commutative algebra: rings, fields, algebras, ideals, modules.
(8) Properties of finite fields.
(9) Nilpotence, radicals, maximal and prime ideals, localization.
(10)* Basic notions of non-commutative algebra. Structure theory for non-commutative algebras.

[4] Algebraic Geometry: A First Geometric Look. Higher school of economics, September-December 2018, 2 hours per week.

- Projective spaces. Geometry of projective quadrics. Spaces of quadrics.
- Lines, conics. Rational curves and Veronese curves. Plane cubic curves. Additive law on the points of cubic curve.
- Grassmannians, Veronese's, and Segre's varieties. Examples of projective maps coming from tensor algebra.
- Integer elements in ring extensions, finitely generated algebras over a field, transcendence generators, Hilbert's theorems on basis and on the set of zeros.
- Affine Algebraic Geometry from the viewpoint of Commutative Algebra. Maximal spectrum, pullback morphisms, Zariski topology, geometry of ring homomorphisms.
- Agebraic manifolds, separateness. Irreducible decomposition. Projective manifolds, properness. Rational functions and maps.
- Dimension. Dimensions of subvarieties and fibers of regular maps. Dimensions of projective varieties.

- Linear spaces on quadrics. Lines on cubic surface. Chow varieties.
- Vector bundles and their sheaves of sections. Vector bundles on the projective line. Linear systems, invertible sheaves, and divisors. The Picard group.
- Tangent and normal spaces and cones, smoothness, blowup. The Euler exact sequence on a projective space and Grassmannian.

Yulij Ilyashenko
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