Topics in explicit algebraic geometry – 2: course proposal

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Program

This course would be a continuation of the course *Topics in explicit algebraic geometry – 1* I taught at IUM in the Spring semester of 2019 where we studied the theory of cubic (and all other del Pezzo) surfaces in detail.

This program might take more than one semester to realize; i.e., the course would continue into the next semester.

Topics in algebraic surfaces – 2

This section contains some explicit examples of algebraic surfaces (and their geometry) which were not studied in the previous course.

- Birational isomorphisms of surfaces. The Cremona group.
- Ruled surfaces, their elementary transformations, and connection with vector bundles of rank two.
- Elliptic surfaces: examples. Degenerate fibers.
- Abelian surfaces: examples.
- K3 surfaces, introduction.
  - Space quartics, and the notion of K3-surfaces.
  - Classical definition of K3 surfaces.
  - Examples of K3 surfaces of genus 2, 3, 4 and 5.
  - Kummer K3 surfaces.
- Some surfaces of general type
  - Surface geography, short introduction.
  - Barlow surface.
  - Some other examples.

- ⋆ Noether's line.
- ⋆ Hirzebruch's examples of surfaces with $c_1^2 = 3c_2$.

- ◦ Some non-classical surfaces in positive characteristics.
- ◦ Introduction to surface singularities.
  - ⋆ Resolution of isolated singularities of surfaces; some examples.
  - ⋆ Du Val singularities and their resolution.

Grassmannian varieties – 2: topics

- ◦ Intersection theory on Gr(2,4): Schubert calculus, and some classical problems of enumerative geometry.

Topics on curves: moduli spaces

- ◦ Curves of genus 5: detailed study, and trisecant lines. Plane models.
- ◦ Severi varieties of plane curves. Dimension of the moduli space of curves: algebraic computation.
- ◦ Moduli space of curves: Idea of the construction; and genus two case.

Topics on curves: space curves

- ◦ Space curves - geography. Halphen's results.
- ◦ Castelnuovo's results.

Topics on curves: vector bundles

- ◦ Vector bundles on curves. Moduli of vector bundles – idea of the construction; and an example: vector bundles on genus 2 curves.

Introduction into the quadratic complex of lines in $\mathbb{P}^3$

- ◦ Linear complex of lines
- ◦ Quadratic complex of lines: definition
- ◦ Associated Kummer surface.
- ◦ Associated K3 surface.
- ◦ Fano variety of X is an abelian surface.