Klein topological field theory and Hurwitz numbers of complex and real algebraic curves

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November 22, 2001

We expand the classical (oriented) $2D$ topological field theory to Klien surfaces. Thus we consider topological field theories for non nessesary oriented surfaces possibly with a boundary and call them Klien topological field theories.

It is well-known that classical two-dimensional topological field theories are in one-to-one correspondence with Frobenius algebras. We prove that Klien topological field theories corresponds to a more complicated algebraic structure which we call a structure algebra. Structure algebra is a non-commutative analogue of a Frobenius algebra. We classify semisimple structure algebras. As an example we construct a stucture algebra for any finite group.

We prove that classical Hurwitz numbers of complex algebraic curves and their analogues for real algebraic curves have natural interpretation in terms of structure constants of a structure algebra for symmetric group.