Bolibruch A.A.
Steklov Mathematical Institute

Semistable Vector Bundles with Connections and the Riemann-Hilbert Problem

We consider the problem of construction of a semistable vector bundle E of zero degree on a compact Riemann surface M equipped with a logarithmic connection ∇ possessing a given monodromy representation

$$\chi : \pi_1(M \setminus \{p_1, \ldots, p_n\}, z_0) \longrightarrow GL(p; C).$$

This problem is a natural generalization (proposed by H.Esnault and E.Viehweg in [1]) of the classical Riemann-Hilbert Problem, because in the case of the Riemann sphere such a bundle E is holomorphically trivial and thus, the connection ∇ defines (in some trivialization of the bundle) a linear system of Fuchsian differential equations with the given singular points and monodromy χ.

We prove (see [2]) the following generalization of the main result from [1].

If the representation χ can be realized as a monodromy of some logarithmic connection ∇' in a vector bundle E' such that the pair (E', ∇') is stable, then the problem mentioned above has a positive solution.

Recall, that a pair (E', ∇') is called *stable* if for every subbundle $F \subset E'$ stabilized by ∇ one has that the slope $\deg(F)/\mathrm{rk}(F)$ of this subbundle is smaller than the slope of the whole bundle E'.

The statement formulated above gives new effective sufficient conditions for the positive solvability of the classical Riemann-Hilbert Problem and also gives rise the following result.

The Riemann-Hilbert Problem has a positive solution in the class of Fuchsian systems with irreducible sets of coefficients if and only if the representation χ can be realized as a monodromy of some logarithmic connection ∇' in a vector bundle E' such that the pair (E', ∇') is stable.

REFERENCES

[1] H. Esnault and E. Viehweg, *Semistable bundles on curves and irreducible representations of the fundamental group*, Providence, RI: Amer. Math. Soc. Contemp. Math. **241** (1999), 129–138.

[2] A.A. Bolibruch, *Stable vector bundles with logarithmic connections and the Riemann-Hilbert problem*, Doklady Mathematics (Proc. of Russian Acad. of Scie.), **381 (2001), no. 1, pp. 1–4 (in russian).**