WHITNEY'S INDEX FORMULA AND LAPLACE INTEGRALS

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Immersions $S^1 \to \mathbb{R}^2$ (i.e. smooth curves in the real plane) are classified by a single invariant, a winding number. It equals the algebraic number of full rotations made by the unit tangent vector of the curve around the origin. Explicitly the winding number of the curve $\gamma(t) = (x(t), y(t))$ is given as

$$\mathrm{wn}(\gamma) = \int \frac{x''(t)y'(t) - y''(t)x'(t)}{\bigl(x'(t)\bigr)^2 + \bigl(y'(t)\bigr)^2}\, dt$$

Whitney's formula relates the winding number of the generic curve with the sum of signs of its self-intersection points.

The talk explains extension of these facts to the case of smooth immersions $S^n \to \mathbb{R}^{2n}$. Such immersions (for n even) are also classified by a single integer, a Smale index; for generic immersions it is equal to the sum of signs of self-intersection points. We provide an explicit integral formula for the index. The proof uses analytical technique of asymptotic expansion of certain Laplace integrals. As a consequence, we get an explicit formula for the generator of de Rham n-th cohomology group of the Stiefel variety $V(n, 2n)$.