Wedderburn-type theorems for operator algebras: traditional and "quantized" homological approaches.

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The classical Wedderburn theorem describes the structure of semi-simple finite-dimensional algebras. These algebras have the nearest natural generalization in the class of operator algebras: von Neumann algebras of type I with discrete center, also called Wedderburn algebras. Many years ago the interest to infinite-dimensional (=functional-analytic) generalizations of Wedderburn Theorem served as one of main stimuli for von Neumann to discover algebras, bearing now his name. Murrey and von Neumann, by inventing continuous factors, have shown that the only assumption on the von Meumann algebra to have the discrete center still does not imply the Wedderburn structure.

We shall show that one can completely describe the Wedderburn algebras in homological terms. An operator algebra A on a Hilbert space H is called *spatially projective* if H is a projective A-module in the sense of traditional Banach homology, and it is called *quantum spatially projective* if H, endowed with the column quantization, is projective as a quantized A-module. Our result is as follows. *Traditionally spatially projective von Neumann algebras are Wedderburn algebras with the additional property of the so-called essential finiteness whereas quantum spatially projective von Neumann algebras are exactly Wedderburn algebras.* As the "crucial" special case, the algebra of all operators on H in the standard form is spatially projective in the sense of quantum but not traditional homological theory.

The structure of spatially projective operator C^*-algebras also can be completely described in the framework of both homological theories. The same is true for the more general question of characterizing projective Hilbert modules over C^*-algebras.