Energy of a knot: variational principles.

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Let $S^1 = \mathbb{R}/(2\pi\mathbb{Z})$ be the circle and $\tau : S^1 \longrightarrow \mathbb{R}^3$ be a knot. Denote by $D(x, y)$ the length of the minimal subarc between x and y on the circle. Let $|*, *|$ be the distance between two points in $\mathbb{R}^3$.

We denote

$$E_f(\tau) = \iint\limits_{S^1\times S^1} f(|\tau(x), \tau(y)|, D(x, y))dxdy$$

the energy of a knot τ for any f from certain class of functions.

The problem is to find knots with extremal values of energy. We discuss possible notions of locally perturbed knots.

Consider the circle. This knot minimizes some energies E_f and maximizes some others. So, is there any energy such that the circle neither maximizes nor minimizes this energy? Recently it was shown (A. Abrams, J. Cantarella, J. H. G. Fu, M. Ghomu, and R. Howard) that the answer is positive. We prove that nevertheless the circle is a locally extremal knot. This means that the circle satisfies certain variational equations. We also present these equations.

Finally we define a new energy, that we call MinMax energy, and describe its main properties.