NONUNIFORM HYPERBOLICITY AND RIGIDITY

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The field of geometric rigidity is concerned with various interesting nonlinear representations of groups, e.g. as fundamental groups of manifolds, or as diffeomorphisms of compact manifolds. The underlying methods involve interplay of Margulis' superrigidity and its derivatives, Gromov' theory of rigid geometric structures, and hyperbolic dynamics. The first ingredient is the most powerful tool but it is only applicable if the groups in question are sufficiently "rigid" themselves. The interplay of superrigidity and the theory of nonstationalry normal forms developed by M.Guysinsky and the author brought about major advances in the understanding of of local rigidity of actions of higher-rank lattices in semisimple Lie groups (A.K–R. Spatzier, G.A. Margulis–N.Qian, V. Nitica–A.Torok, G.A.Margulis–D. Fisher) These developments will be briefly surveyed in the first part of the talk.

The main part of the talk will deal new global results. They are concerned with actions of higher–rank abelian groups on compact manifolds. Since those groups are amenable no purely measurable general tools like cocycle superrigidity are available. Hyperbolic dynamics steps in to compensate for this deficiency. For the first time now there are results which require no a priori dynamics assumptions. A special result which can serve as a model for more general situations is the following.

Theorem. *An action Φ of $\mathbb{Z}^2$ on $\mathbb{T}^3$ by C^2 diffeomorphisms, homotopic to a hyperbolic (Cartan) linear action Φ_0 always preserves an absolutely continuous ergodic measure μ. Moreover, there is unique continuous absolutely continuous semiconjugacy h between Φ and Φ_0 and μ is the only measure which is mapped to Lebesgue measure.*

Needless to say, this result presents a sharp contrast to the rank one case where even for an Anosov map are close to a linear the topological conjugacy with the linear model is usually singular.