Gelfand–Tsetlin schemes, measures of hypergeometric type, and point processes. Talk by Grigori Olshanski

Dobrushin Math. Lab., Institute for Information Transmission Problems, Moscow

The subject of my talk is a remarkable family $\{P_N\}$ of probability measures depending on four parameters. Here $N = 1, 2, \dots$, and the Nth measure P_N lives on the space of dominant weights of the compact unitary group $U(N)$. This space of weights can be viewed as a subset of the lattice $\mathbb{Z}^N$. In particular, P_1 is a probability measure on $\mathbb{Z}$.

The family $\{P_N\}$ originated in the study of representations of the infinite–dimensional unitary group $U(\infty)$, see Olshanski [O]. On the other hand, the measures P_N can be obtained by a direct combinatorial construction based on Gelfand–Tsetlin schemes. This construction is rather elementary, and I will explain it.

One of the beautiful formulas in the classical theory of special functions is Dougall's formula

$$\sum_{k\in\mathbb{Z}} \frac{1}{\Gamma(z-k+1)\Gamma(z'-k+1)\Gamma(w+k+1)\Gamma(w'+k+1)}$$
$$= \frac{\Gamma(z+z'+w+w'+1)}{\Gamma(z+w+1)\Gamma(z+w'+1)\Gamma(z'+w+1)\Gamma(z'+w'+1)},$$

which can be viewed as a two–sided analog of the famous Gauss summation formula for the hypergeometric function.

Our first measure P_1 is closely related to Dougall's formula: the expression for $P_1(k)$ is obtained by dividing the kth term of the left–hand side by the right–hand side. The explicit expression for the higher measures P_N provides a multivariate generalization of Dougall's formula.

Note also that P_N can be interpreted as a N–particle log–gas system on $\mathbb{Z}$.

The most interesting and important question is as follows: what happens with our measures in a large–N limit transition. It turns out that the limit object $P = \lim_{N\to\infty} P_N$ exists and can be interpreted as a continuous point process. In other words, the limit P is again a probability measure, but its carrier space is no longer discrete. It is an infinite–dimensional space whose elements are certain infinite point configurations on $\mathbb{R}$.

An explicit description of the point process P is obtained in Borodin–Olshanski [BO]. This result also provides a solution to a problem of harmonic analysis on the group $U(\infty)$.

References

[BO] A. Borodin and G. Olshanski, *Harmonic analysis on the infinite–dimensional unitary group and determinantal point processes*, Preprint, 2001, 88 pp., `http://arXiv.org/abs/math.RT/0109194`.

[O] G. Olshanski, *The problem of harmonic analysis on the infinite–dimensional unitary group*, Preprint, 2001, 50 pp., `http://arXiv.org/abs/math.RT/0109193`.