Dynamical systems which admit periodical decompositions.

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By a *dynamical system* (d. s. in what follows) we mean a pair (X, f) where X is a compact Hausdorff space and $f : X \to X$ is a homeomorphism of X onto itself.

Let (X, f) and (Y, g) be two d. s.. A continuous map $h : X \to Y$such that $h(X) = Y$ and $h \circ f = g \circ h$ will be called a *projection* of (X, f) onto (Y, g) and will be denoted by $h : (X, f) \to (Y, g)$.

An *odometer* is a d. s. induced by a group rotation over an adic group. Equivalently, odometer is a distal (equicontinuous) minimal d. s. with a Cantor set as a phase space.

We investigate curtain properties of d. s. which admit a projection onto an odometer.

Let (X, f) be a d. s.. We call a decomposition of X into a finite family $\{W_i^{(n)}\}_{i=0}^{n-1}$ of disjoint closed subsets by a *periodical decomposition* of (X, f) if $f(W_{n-1}^{(n)}) = W_0^{(n)}$ and $f(W_{i-1}^{(n)}) = W_i^{(n)}$, $i = 1, \dots, n-1$. The number n of sets in decomposition is called its *length*. Denote by $\mathcal{P}(X, f)$ the set of all lengths of periodical decompositions.

It is claimed that a d. s. (X, f) admits a projection onto an odometer if and only if the set $\mathcal{P}(X, f)$ is of infinite cardinality. If a d. s. (X, f) admits a projection onto an odometer then on the set $\mathcal{A}$ of all odometers onto which (X, f) could be projected we can introduce natural partial order and the ordered set $(\mathcal{A}, \leq)$ has a greatest element. Moreover, if d. s. (X, f) is *indecomposable* (i. e. if the space X could not be presented as a disjoint union of two nonempty closed f-invariant subsets) then a projection onto a greatest odometer has the property of universality in the category of all projections of (X, f) onto odometers.

It is claimed also that a d. s. (X, f) admits an almoust one-to-one projection onto an odometer if and only if (X, f) is an almoust periodic d. s..