THE SECOND ORDER CASIMIRS FOR THE AFFINE KRICHEVER-NOVIKOV ALGEBRAS $\widehat{\mathfrak{gl}}_{g,2}$ AND $\widehat{\mathfrak{sl}}_{g,2}$

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The second order casimirs for affine Krichever-Novikov algebras $\widehat{\mathfrak{gl}}_{g,2}$ and $\widehat{\mathfrak{sl}}_{g,2}$ are described. More general operators which we call semi-casimirs are introduced. It is proved that semi-casimirs induce well-defined operators on conformal blocks and there is a natural projection of $T_\Sigma\mathcal{M}_{g,2}^{(1)}$ onto the space of these operators where $\mathcal{M}_{g,2}^{(1)}$ is the moduli space of Riemann surfaces with two marked points and fixed 1-jets of local coordinates at those points, $\Sigma\in\mathcal{M}_{g,2}^{(1)}$ is the Riemann surface involved into the definition of Krichever-Novikov algebras in question.

Let Σ be a compact algebraic curve over $\mathbb{C}$ of genus g with two marked points $P_\pm$, $\mathcal{A}$ be the algebra of meromorphic functions on Σ which are regular exept at the points $P_\pm$, $\mathfrak{g}$ be a complex semi-simple or reductive Lie algebra. Then

$$\widehat{\mathfrak{g}}=\mathfrak{g}\otimes_{\mathbb{C}}\mathcal{A}\oplus\mathbb{C}c$$

is called the *affine Krichever-Novikov algebra* (first introduced in [1]). The bracket on $\widehat{\mathfrak{g}}$ is given by the formula

$$[x\otimes A,y\otimes B]=[x,y]\otimes AB+\gamma(x\otimes A,y\otimes B)c,\ \ [x\otimes A,c]=0,$$

where

$$\gamma(x\otimes A,y\otimes B)=\operatorname{tr}(\operatorname{ad}x\circ\operatorname{ad}y)\operatorname{res}_{P_+}(AdB).$$

Let also Ł be the algebra of meromorphic vector fields on Σ with the same analitic properties and $\mathcal{D}^1$ be the semi-direct sum of $\widehat{\mathfrak{g}}$ and Ł.

In [2] the fermion (or wedge) representations of the affine Krichever-Novikov algebras were introduced. They are parametrized by the triples consisting of a holomorphic vector bundle on Σ, a dominant weight of $\mathfrak{g}$ and of an integer (*the charge*). There are two actions of Ł in the space of a fermion representation. The first is due to the existence

* Supported in part by RFBR Project 99-01-00198

of a meromorphic (therefore flat) logarithmic conection on the corresponding bundle (for $e \in \mathrm{L}$ we denote this action $\widehat{e}$). The second is the Sugawara representation of L (for $e \in \mathrm{L}$ we denote it $T(e)$). Consider operators of the form $\Delta(e) = \widehat{e} - T(e)$. All these actions can be continued on a certain completion $\overline{\mathrm{L}}$ of L. Observe that $[\Delta(e), \widehat{\mathfrak{sl}}_{g,2}] = 0$ for any $e \in \overline{\mathrm{L}}$.

Definition 1. $\Delta(e)$ is called a (second order) casimir for $\widehat{\mathfrak{g}}$ iff $[\Delta(e), \mathcal{A}] = 0$.

Theorem 1. *For a generic fermion representation there is exactly one (up to a scalar factor) casimir. The corresponding vector field has a simple zero at the point P_+.*

Definition 2. $\Delta(e)$ is called a semi-casimir for $\widehat{\mathfrak{g}}$ iff $[\Delta(e), \tilde{\mathcal{A}}_-] = 0$ where $\tilde{\mathcal{A}}_-$ denotes the subspace of "generating" operators in $\mathcal{A}$.

Theorem 2. *Semi-casimirs induce well-defined operators on the space of conformal blocks. For a generic fermion representation there is a natural projection of $T_\Sigma \mathcal{M}_{g,2}^{(1)}$ onto the space of operators induced by semi-casimirs on conformal blocks over Σ.*

References

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