Combinatorial formulas for cohomology of spaces of knots

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Any cohomology class of the space of knots in $\mathbb{R}^n$, $n \geq 3$ (in particular any numerical invariant of knots in $\mathbb{R}^3$) can be realized as the linking number with appropriate infinite-dimensional cycles in the corresponding space Σ of singular curves. A *combinatorial formula* for the cohomology class α is an *easy* (in some precise sense) semialgebraic chain in the space of all parametrized spatial curves, whose boundary lies in Σ and equals such a cycle realizing α.

We propose an efficient method of computing such combinatorial formulas for finite-type cohomology classes of the space of knots. This method is *purely combinatorial*, i.e. it deals not with the planar pictures and their deformations, but with easily encodable objects generalizing the chord diagrams and the (Polyak–Viro) arrow diagrams. In the most classical case of knot invariants our algorithm is a chain of linear algebraic operations over these objects, similar to (and starting with) checking the homological 4T- and 1T-conditions for a sum of chord diagrams. In particular, the complexity of the algorithm and of its answers depends only on the order of the invariant, and not on the complexity of arising knot diagrams.

The formulas obtained in this way allow one to prove the existence of some cohomology classes predicted previously by algebraic methods, e.g. of the *Teiblum–Turchin cocycle* (which is the first positive-dimensional cohomology class of finite degree).

The methods exploit a deep analogy between the theory of knot invariants and that of affine plane arrangements.

References

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