The dual horospherical Radon transform for polynomials

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Let $X = G/K$ be a semisimple symmetric space of the non-compact type. A horosphere in X is an orbit of a maximal unipotent subgroup of G. The set $\mathrm{Hor}\,X$ of all horospheres is a homogeneous space of G. The horospherical Radon transform R suggested by I.M.Gelfand and M.I.Graev in 1959 takes a function ϕ on X to a function on $\mathrm{Hor}\,X$ obtained by integrating ϕ over horospheres.

The manifolds X and $\mathrm{Hor}\,X$ can be naturally considered as real algebraic varieties. Thereby the algebras $\mathbb{R}[X]$ and $\mathbb{R}[\mathrm{Hor}\,X]$ of polynomial (=regular rational) functions are defined. The natural linear representations of G in $\mathbb{R}[X]$ and $\mathbb{R}[\mathrm{Hor}\,X]$ are isomorphic to each other and multiplicity-free.

The transform R is not defined for polynomials but its dual transform R^* is defined. Since it is G-equivariant, it takes any irreducible component of $\mathbb{R}[\mathrm{Hor}\,X]$ to the isomorphic component of $\mathbb{R}[X]$. This determines R^* up to some factors $C(\lambda)$ where λ runs over the heighest weights of irreducible components of $\mathbb{R}[X]$. The main result of the talk is that

$$C(\lambda) = c(\lambda + \rho)$$

where c is the well-known Harish-Chandra c-function and ρ is the half-sum of positive roots of X. This is a joint result with J.Hilgert and A.Pascuale.