ASSIGNMENT 1

Problem 1.1. Prove that any functor $T : \mathcal{C} \to \mathcal{D}$ assigns isomorphic objects to isomorphic objects.

Problem 1.2. In an arbitrary category define the notion of right inverse and left inverse of a morphism $f : X \to Y$ and show that if they both exist, then they coincide and f is an isomorphism. Prove that any isomorphism has a unique inverse.

Problem 1.3. Define the inverse limit in an arbitrary category by using the category $\mathcal{D}ir$

In order to solve the subsequent problems, you can assume known the homology groups of $\mathbb{D}^n, \mathbb{S}^n, \mathbb{R}P^n, \mathbb{C}P^n, \mathbb{T}^2, M_g^2$.

Problem 1.4. Is there a retraction of $\mathbb{S}^5$ on $\mathbb{S}^4$, where $\mathbb{S}^4$ is the "equator" of $\mathbb{S}^5$, i.e. $\mathbb{S}^5 = \Sigma\mathbb{S}^4$ (here Σ denotes suspension) ?

Problem 1.5. Is there a retraction of $\mathbb{R}P^5$ on $\mathbb{R}P^4$, where $\mathbb{R}P^4 \hookrightarrow \mathbb{R}P^5$ is the natural embedding given by $(x^1; \ldots; x^5) \mapsto (x^1; \ldots; x^5; 0)$?

Problem 1.6. Is there a retraction of the solid torus $S = \mathbb{S}^1 \times \mathbb{D}^2$ on its boundary $dS = \mathbb{T}^2 = \mathbb{S}^1 \times \mathbb{S}^1$?

Problem 1.7. Given a map $p : \mathbb{R}P^2 \to \mathbb{S}^1 \times \mathbb{D}^{17}$ and a homeomorphism $h : \mathbb{S}^1 \times \mathbb{D}^{17} \to \mathbb{S}^1 \times \mathbb{D}^{17}$, is it possible to lift h to a map $H : \mathbb{S}^1 \times \mathbb{D}^{17} \to \mathbb{R}P^2$?

Problem 1.8. Prove that Euclidean spaces of different dimensions are not homeomorhic.

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