ASSIGNMENT 2

Problem 2.1. Let C be the union of all circles of center $(\frac{1}{n}, 0)$ in the xy-plane and radius $\frac{1}{n}$. Prove that C is not homeomorphic to a CW-space.

Problem 2.2. a) Find a space satisfying (W) but not (C).
b) Find a space satisfying (C) but not (W).
c) Is the closure of cell necessarily a subspace ?

Problem 2.3. Find mimimal CW-complex structure on $\mathbb{C}P^n$, $\mathbb{R}P^n$, $\#_{i=1}^q \mathbb{T}_i^2$, $\#_{i=1}^q \mathbb{R}P_i^2$.

Problem 2.4. Define $\mathbb{S}^\infty$, $\mathbb{R}P^\infty$, $\mathbb{C}P^\infty$ and supply them with CW-structure.

Problem 2.5. Prove that $\mathbb{S}^\infty$ is contractible.

Problem 2.6. Give an example of a nontriangulable two-dimensional CW-complex.

Problem 2.7. Prove that $\pi_1 X$ lives in $X^{(2)}$ (the 2-skeleton of X), i. e., $\pi_1(X) \equiv \pi_1(X^{(2)})$.

Problem 2.8. Prove that a CW-complex is connected iff its 1-skeleton $X^{(1)}$ is connected.

Problem 2.9. Prove that a CW-complex is connected iff it is path-connected.

Problem 2.10. Prove that $\pi_k \mathbb{S}^n = 0$ for all $k < n$.

Problem 2.11. Show that the Cartesian product, the cone the suspension, the join of CW-complexes have natural CW-complexes structures.

Problem 2.12. Any finite CW-complex X^n can be embedded in $\mathbb{R}^N$, $N = \frac{(n+1)(n+2)}{2}$.

Problem 2.13. Show that $\pi_n(\mathbb{S}^1)$ are trivial for all $n > 1$. (Hint : Consider the bundle $p : \mathbb{R}^1 \to \mathbb{S}^1$ whose projection is given by formula $p(x) = \exp(xi)$)

Problem 2.14. Prove that a locally trivial fibration $p : \mathbb{S}^n \to B$ whose base B consists of more than one point is not homotopic to a constant map.