ASSIGNMENT 3

Problem 3.1. $\pi_k X$ lives in $X^{(k+1)}$, more precisely $i_k : X^{(k)} \hookrightarrow X$ induces an isomorphism $(i_k)_* : \pi_k X^{(k+1)} \to \pi_k X$ for $k < n$ and an epimorphism for $k = n$ (n is the dimension of X)

Problem 3.2.

$$\pi_n(\mathbb{S}^1 \vee \mathbb{S}^n) = \mathbb{Z} \oplus \mathbb{Z} \oplus \cdots \oplus \mathbb{Z} \oplus \dots$$

Problem 3.3. Prove that the product of spheroids is well defined.

Problem 3.4. Prove that the induced homomorphism $f_* : \pi_n X \to \pi_n Y$ is well defined

Problem 3.5. Prove the exactness of the homotopy sequence for fiber bundles at the other terms.

Problem 3.6. Prove the exactness of the homotopy sequence for pairs at the other terms.

Problem 3.7. Prove that $\mathbb{C}P^2 = \mathbb{D}^4 \cup \mathbb{C}P^1$, where $p : \mathbb{S}^3 \to \mathbb{C}P^1$ is the Hopf bundle and $\mathbb{S}^3 = \partial\mathbb{D}^4$.

Problem 3.8. Does there exist a retraction $r : \mathbb{C}P^2 \to \mathbb{C}P^1$, where $\mathbb{C}P^1$ is embedded in $\mathbb{C}P^2$ in the natural way.

Problem 3.9. If A is a retract of X, then

a) the map $i_* : \pi_n(A) \to \pi_n(X)$ is injective ;
b) the map $p_* : \pi_n(X) \to \pi_n(X, A)$ is surjective;
c) the map $\partial_* : \pi_n(X, A) \to \pi_{n-1}(A)$ is a zero homomorphism;
d) $\pi_n(X) = \pi_n(X, A) \oplus \pi_n(A)$.

Problem 3.10. If there exists a homotopy $f_t : X \to X$ such that $f_0 = id$ and $f_1(X) \subset A$ then

a) the map $i_* : \pi_n(A) \to \pi_n(X)$ is surjective ;
b) the map $p_* : \pi_n(X) \to \pi_n(X, A)$ is a zero homomorphism;
c) the map $\partial_* : \pi_n(X, A) \to \pi_{n-1}(A)$ is injective;
d) $\pi_n(A) = \pi_{n+1}(X, A) \oplus \pi_n(X)$.