ASSIGNMENT 4 (CELLULAR HOMOLOGY)

Problem 4.1. Compute $H_*(M_g^2)$, where M_g^2 is the sphere with g handles.

Problem 4.2. Compute $H_*(N_g^2)$, where N_g^2 is $\mathbb{R}P^2$ with g handles.

Problem 4.3. Compute $H_*(\mathbb{C}P^n)$.

Problem 4.4*. Compute $H_*(\mathbb{R}P^n)$.

Problem 4.5. Compute $H_*(\Gamma)$, where Γ is the following graph.

Problem 4.6. Let p and q be two relatively prime positive integers. Consider the action of the group $\mathbb{Z}_p$ with the generator σ on the unit sphere $\mathbb{S}^3 \subset \mathbb{C}^2$ defined by

$$\sigma(z, w) = (\exp(\frac{2\pi i}{p}), \exp(\frac{2\pi i q}{p})).$$

The quotient of $\mathbb{S}^3$ by this action is a 3-manifold. It is called *a lens space* and is denoted by $L(p, q)$. Compute $H_*(L(p, q)$

Problem 4.7. Compute $H_*(\mathcal{D})$, where $\mathcal{D}$ is the dunce hat, i.e., the triangle with the identifications shown by arrows.

Problem 4.8. Compute $H_*(\bar{S})$, where $\bar{S}$ is the sphere $\mathbb{S}^2$ to which the segment $[NS]$ joining the North and South poles have been added.

Problem 4.9. Express $H_n(A \vee B)$ in terms of $H_n A$ and $H_n B$, $n = 0, 1, \ldots$

Problem 4.10. Is it possible to retract the dunce hat to its circle NM ?