ASSIGNMENT 5 (SIMPLICIAL HOMOLOGY)

Problem 5.1. Consider the chain complex

$$0 \xrightarrow{\partial_4} \mathbb{Z} \xrightarrow{\partial_3} \mathbb{Z} \xrightarrow{\partial_2} \mathbb{Z} \xrightarrow{\partial_1} 0$$

where $\partial_3(z) = 2z$ and $\partial_2(z) = 0$. Compute its homology.

Problem 5.2. Compute the homology of the singleton and the segment $[0, 1]$.

Problem 5.3. Compute the homology of the subdivided segment

Problem 5.4. Compute the homology of the boundary of the triangle and the boundary of the square. Compare.

Problem 5.5. Compute the homology of the boundary of the tetrahedron and the boundary of the cube (its faces are triangulated by means of their diagonals). Compare.

Problem 5.6. Prove that the induced homomorphism is well defined.

Problem 5.7. Prove the Poincaré lemma in details (check that the two simplices $[v_0, \ldots, \check{v}_j, \ldots, \check{v}_i, \ldots, v_n]$ do appear with opposite signs).

Problem 5.8. Prove that a simplicial space X is connected if and only if $H_0 X = \mathbb{Z}$.

Problem 5.9. Compute the homology groups $H_*(M\ddot{o}b, \mathbb{Z})$ of the Möbius strip.