PROBLEMS for Lecture 4

4.1. Prove that any motion of the plane is either a translation by some vector v, $|v| \geq 0$, or a rotation r_A about some point A by a nonzero angle.

4.2. Prove that any orientation-reserving isometry of the plane is a glide reflection in some line L with glide vector u, $|u| \geq 0$, $u||L$.

4.3. Justify the following construction of the composition of two rotations $r = (a, \varphi)$ and (b, ψ). Join the points a and b, rotate the ray $[a, b\rangle$ around a by the angle $\varphi/2$, rotate the ray $[b, a\rangle$ around b by the angle $-\psi/2$, and denote by c the intersection point of the two obtained rays; then c is the center of rotation of the composition rs and its angle of rotation is $2(\pi - \varphi/2 - \psi/2)$. Show that this construction fails in the particular case in which the two angles of rotation are equal but opposite, and then their composition is a parallel translation).

4.4. Prove that the composition of a rotation and a parallel translation is a rotation by the same angle and find its center of rotation.

4.5. Prove that the composition of two reflections in lines l_1 and l_2 is a rotation about the intersection point of the lines l_1 and l_2 by an angle equal to twice the angle from l_1 to l_2.

4.6. Indicate a finite system of generators for the transformation groups corresponding to each of the tilings shown in Figure 4.4 a), b),...,f).

4.7. Is it true that the transformation group of the tiling shown on Figure 4.4 (b) is a subgroup of the one of Figure 4.4 (c)?

4.8. Indicate the points that are the centers of the rotation subgroups of the transformation group of the tiling shown in Figure 4.4(c).

4.9. Write out a presentation of the isometry group of the plane preserving
(a) the regular triangular lattice;
(b) the square lattice;
(c) the hexagonal (i.e., honeycomb) lattice.

4.10. For which of the five Platonic bodies can a (countable) collection of copies of the body fill Euclidean 3-space (without overlaps)?

4.11. For the two Escher pictures in Fig.4.2 indicate to which of the 17 Fedorov groups they correspond.

4.12. Exactly one of the 17 Fedorov groups contains a glide reflection but no reflections. Which one?

4.13. Which two of the 17 Fedorov groups contain rotations by $\pi/6$?

4.14. Which three of the 17 Fedorov groups contain rotations by $\pi/2$?

4.15. Which five of the 17 Fedorov groups contain rotations by π only?

4.16. Rearrange the question marks in the tiling (c) so as to make the corresponding geometry isomorphic that of the tiling (a).