PROBLEMS for Lecture 5

In all the problems below a, b, c are the sides and α, β, γ are the opposite angles of a spherical triangle. The radius of the sphere is $R = 1$.

6.1. Prove the first cosine theorem on the sphere $\mathbb{S}^2$:
$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha$.

6.2. Prove the second cosine theorem on the sphere $\mathbb{S}^2$:
$\cos \alpha + \cos \beta \cos \gamma = \sin \beta \sin \gamma \cos a$.

6.3. Prove that $a + b + c < 2\pi$.

6.4. Does the Pythagorean theorem hold in spherical geometry? Prove the analogs of that theorem stated in Corollary 6.5.3.

6.5. Does the Moscow–New York flight fly over Spain? Over Greenland? Check your answer by stretching a thin string between Moscow and NY on a globe.

6.6. Find the infimum and the supremum of the sum of the angles of an equilateral triangle on the sphere.

6.7. The city A is located at the distance 1000km from the cities B and C, the trajectories of the flights from A to B and from A to C are perpendicular to each other. Estimate the distance between B and C. (You can take the radius of the Earth equal to 6400km)

6.8*. Find the area of the spherical disk of radius r (i.e., the domain bounded by a spherical circle of radius r).

6.9. Find fundamental domains for the actions of the isometry groups of the tetrahedron, the cube, the dodecahedron, and the icosahedron on the 2-sphere and indicate the number of their images under the corresponding group action.

6.10. Prove that any spherical triangle has a circumscribed and an inscribed circle.

6.11. Prove that the medians of a spherical triangle intersect at one point.

6.12. Prove that the altitudes of a spherical triangle always intersect at one point.

6.13. Suppose that the medians and the altitudes of a spherical triangle interest at the points M and A respectively. Can it happen that $M = A$?