A PL EMBEDDING OF THE 2-SPHERE IN $\mathbb{R}^4$ WITH NON-STANDARD REGULAR NEIGHBORHOOD

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Abstract. We construct a (non-locally flat) PL embedding $S^2 \subset \mathbb{R}^4$ such that the regular neighborhood of S^2 in $\mathbb{R}^4$ is not homeomorphic to $S^2 \times D^2$. This example is motivated by the following general problem: find conditions on a k-polyhedron K and PL m-manifold M under which for each two homotopic embeddings $f, g : K \to M$, the regular neighborhoods in M of $f(K)$ and $g(K)$ are homeomorphic. Our example is based on construction of a knot, concordant to the trivial one but (with any frame) not Kirby-equivalent to the trivial knot with the frame 0.