ON THE GENERALIZED MASSEY–ROLFSEN INVARIANT FOR LINK MAPS

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Abstract. For $P = P_1 \sqcup \cdots \sqcup P_s$ and a link map $f : P \to \mathbb{R}^m$ let $\overline{P} = \sqcup_{i<j} P_i \times P_j$, define a map $\widetilde{f} : \overline{P} \to S^{m-1}$ by $\widetilde{f}(x,y) = \frac{fx-fy}{|fx-fy|}$ and a (generalized) Massey–Rolfsen invariant $\alpha(f) \in \pi^{m-1}(\overline{P})$ to be the homotopy class of $\widetilde{f}$. We prove that for a polyhedron P of dimension $\leq m-2$ under certain (weakened metastable) dimension restrictions, α is an onto or a 1–1 map from the set of link maps $f : P \to \mathbb{R}^m$ up to link concordance to $\pi^{m-1}(\overline{P})$. We present calculation of $\pi^{m-1}(\overline{P})$ for some cases. If $P_1, \dots, P_s$ are closed highly homologically connected manifolds (in particular, homology spheres), then $\pi^{m-1}(\overline{P}) \cong \oplus_{i<j} \pi^S_{p_i+p_j-m+1}$. By the theorem of Melikhov, 'link concordance' can be replaced by 'link homotopy' for codimension at least 3.

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