CURRICULUM VITAE

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AREA OF SPECIALIZATION:
– 3-Dimensional topology;
– Geometry, singularity theory.

EDUCATION:
Ph. D., Apr. 1998 Mathematics, Moscow State University (MGU) / Independent University of Moscow. Advisor: V. A. Vassiliev.
M. S., 1993 Mathematics, Moscow State University (MGU).

SELECTED AWARDS:
2001 Moscow Mathematical Society Prize (for the paper "Towards lower bounds for complexity of 3-manifolds: a program")
2000–2002 EPDI Fellowship (the best postdoc opportunity in Europe)
Sep.–Dec. 1998 Membership of the School of Mathematics, Institute for Advanced Study, Princeton;
1986 First Prize of the All-Union Mathematical Olympiad.

PROFESSIONAL EXPERIENCE:
2000–now EPDI (*European PostDoctoral Institute*) Fellowship (my program includes I. Newton Institute, Cambridge, UK, IHES, Bures-sur-Yvette, France, Université Joseph Fourier, Grenoble, France, and Utrecht University, Utrecht, the Netherlands);
1999–2000 Postdoctoral fellow, Instituto Superior Técnico, Lisbon, Portugal
Jan.–Apr. 1999 Participant of the program "Topology and Geometry of Quantum Fields", Mittag–Leffler Institute, Djursholm, Sweden;
Sep.–Dec. 1998 Member of the School of Mathematics, *Institute for Advanced Study*, Princeton, USA;
Jan. 1998–now Assistant Professor, Mathematics College, Independent University of Moscow;
1993–1997 Teaching Assistant, Mathematics College, Independent University of Moscow.

SUMMARY OF TEACHING EXPERIENCE:

In 1989–1993 I taught Soviet (since 1992, Russian) team for International Mathematical Olympiads.

In 1993–1997, I was a Teaching Assistant at the Independent University of Moscow. Below is the list of courses:
Differential Geometry (following lecture courses by S. P. Novikov and S. M. Gusein-Zade),Partial Differential Equations (following V. I. Arnold), Topology (following A. B. Sossinsky and V. A. Vassiliev), Ordinary Differential Equations (following Yu. S. Ilyashenko), and Geometry (following Yu. M. Burman).

Since January 1998, I am an Assistant Professor at the Independent University of Moscow. During the spring semester 1998, I taught a course in Topology.

SELECTED LECTURES AT INTERNATIONAL CONFERENCES:

December 2000 "Applications of Singularity Theory to Geometry", Cambridge–Liverpool, United Kingdom.
June 1998 "Geometry and Topology of Caustics", S. Banach Mathematical Center, Warsaw, Poland.
August 1996 "Real and Complex Singularities and Their Applications (Conference dedicated to C. T. C. Wall's 60th birthday)", Liverpool, United Kingdom.
May 1996 "Topology and Applications (International topological conference dedicated to P. S. Aleksandrov's 100th birthday)", Moscow State University, Moscow, Russia;
October 1992 "International Conference on Real Algebraic Geometry, Symplectic Topology, and Singularity Theory", Euler Institute, S.-Petersburg, Russia;
June 1992 "Workshop on Dynamical Systems", International Centre for Theoretical Physics, Trieste, Italy;

PERSONAL INFORMATION:

Born: March 14, 1972, Khar'kov, USSR
Citizenship: Citizen of Russia
Languages: English, French; Russian (native), Portuguese (basic).

PUBLICATIONS (for details, see Research Summary below):

[1] *On the topology of integrable Hamiltonian systems in the neighborhood of 1-dimensional nonsingular trajectory*, Trudy MIRAN (Proc. V. A. Steklov Mathematics Institute), Vol. 205 (1994), 4–11.

[2] *The automorphism group of Hilbert's example of projective plane*, Uspekhi Mat. Nauk (Russian Math. Surveys), Vol. 47 (1992), issue 3 (285), 147–148.

[3] *Flip-equivalence of triangulations of surfaces*, Vestnik Moskov. Univ. (Bulletin of the Moscow State University), Ser. 1 (Mathematics, mechanics) (1994) issue 2, 61–67.

[4] *Projective convex curves*, in: *Arnold–Gelfand Mathematical Seminars: Geometry and Singularity Theory*, Eds.: V. I. Arnold, I. M. Gelfand, V. S. Retakh, and M. M. Smirnov. Birkhäuser, Boston (1997), 93–99.

[5] *"Area–length" duality and the characteristic* 2*-chain*, Mat. Zametki (Math. Notes) Vol. 58 (1995), issue 3, 445–446.

[6] *Convex curves in* $\mathbb{R}\mathbf{P}^n$, Trudy MIRAN (Proc. V. A. Steklov Mathematics Institute), Vol. 221 (1998), 3–39.

[7] (joint with S. K. Lando) *Topological complexity of torus bundles over* S^1, in: Topics in Quantum Groups and Finite–type Invariants (Mathematics at the Independent University of Moscow) (AMS Translations ser. 2, Vol. 185; Advances in the Mathematical Sciences). Eds.: B. L. Feigin, V. A. Vassiliev (1998), 129–135.

[8] *Integral formulas related to wave fronts*, in: Geometry and Topology of Caustics — Caustics-98, Banach Center Publications, Vol. 43, 11–17 (1999).

[9] (Editor of the brochure) *LXI Moscow Mathematical Olympiad. Problems and Solutions.* Publ.: Moscow Center for Continuous Mathematical Education (1998).

PREPRINTS:

[10] *Upper bounds for complexity of some* 3*-dimensional manifolds*, I. Newton Institute preprint NI00037-SGT, 2000.

[11] *Towards lower bounds for complexity of* 3*-manifolds: a program*, IHES preprint M/01/13; also available through `http://xxx.lanl.gov` as math. GT/0103169.

[12] *Exact values of complexity for an infinite number of* 3*-manifolds*, in preparation.

RESEARCH SUMMARY (for a description of my recent recearch, see the "Research in progress" section below):

In my first paper [1] normal forms of the Hamilton function and of the Poincaré mapping are applied to the study of some questions of topological classification of integrable Hamiltonian systems on 4-dimensional manifolds.

In [2] one particular example of projective plane is considered. An ellipse $x^2/a^2 + y^2/b^2 = z^2$ cuts the standard projective plane into two parts. One of them is homeomorphic to the Möbius band, and the lines in it are defined in the usual way. In the second part, lines are defined as circles going through $M = (0, 1)$. Obviously, the reflection in the Ox axis is an automorphism of this plane. It is proved in the paper that it is the only nontrivial automorphism.

Since 1994, I work in singularity theory. My thesis "Problems of topology, integral geometry, and complexity in singularity theory" is based on the results published in [3–7].

In [5] I proved that the oriented length $c(t)$ of a wave front that moves in S^2 with unit velocity and the area $s(t)$ of the so-called characteristic 2-chain (a certain 2-chain bounded by this front) satisfy the differential equations $s' = c$ and $c' = -s$. Thus $c(t) = a\cos(t + b)$ and $s(t) = a\sin(t + b)$. This provides a simple proof of the area-length duality, which states that $c(t) = s(t + \pi/2)$ and $s(t) = -c(t + \pi/2)$. Furthermore, this leads to a clearer definition of the characteristic 2-chain.

The study of this topic was continued. Later I showed that for the n-dimensional sphere, the oriented $(n-1)$-dimensional volume of a wave front is a polynomial in $\sin t$ and $\cos t$ of degree $n-1$, while for the Lobachevskii space, $c(t)$ is a polynomial in $\cosh t$ and $\sinh t$ of degree $n-1$ (see [8]). This results seem to be similar to Weyl's results on the volume of tubes (for example, see A. Gray, *Tubes*). However, Weyl (and Gray) considered only "nonoriented" volumes.

The paper [7] is devoted to multidimensional generalizations of Kazarian's results on S^1-bundles, relative Morse theory, and cyclic cohomology. The goal of [7] is to obtain similar results for bundles with multidimensional fibers. In [7] we consider T^2 bundles over S^1. We define the topological complexity of a bundle of this kind as the minimal possible number of standard bifurcations occurring in a family of θ-curves embedded into fibers; a θ-curve here is the 1-skeleton of a CW-decomposition of T^2 with only one 2-cell. This complexity $c(A)$ (where $A \in \mathrm{SL}(2, \mathbb{Z})$ is the monodromy of the bundle) is well defined. Moreover, it is equal to the number of bifurcations (taken with their signs), because the operator A determines a coorientation of the space of degenerate θ-curves (i.e., θ-curves with one edge contracted), which forms a discriminant in the space of all θ-curves. Finally,we obtain an explicit expression for $c(A)$ in terms of the denominators ofa certain continued fraction associated with A.Some definitions and proofs in [7] are based on the result of [3].

Probably, the most referred of my pre-PhD papers is [6] (some results of [6] had been announced in [4]). A closed curve in $\mathbb{R}\mathbf{P}^n$ is *convex* if any hyperplane intersects it at most n times, counting multiplicities. Other facets of this object are Chebyshev systems of functions and disconjugate periodic differential equations with scalar monodromy operator. Some of my results here are as follows.

1. *The space of all parameterized convex curves is homotopy equivalent to either* $\mathrm{SO}(n+1, \mathbb{R})$ *or* $\mathrm{SO}(n+1, \mathbb{R})/\pm \mathrm{Id}$, *depending on the parity of* n.
2. *For* $A \in \mathbb{R}\mathbf{P}^{n+1} \setminus \mathbb{R}\mathbf{P}^n$, *any convex curve* $\gamma \subset \mathbb{R}\mathbf{P}^n$ *is the central projection from* A *of some convex curve* $\Gamma \subset \mathbb{R}\mathbf{P}^{n+1}$ *that passes through* A.
3. (V. I. Arnold's conjecture) *Any convex curve in* $\mathbb{R}^{2n}$ *has a convex projection on*

some subspace $\mathbb{R}^{2n-2}$.

Brochure [9] contains the problems proposed for LXI Moscow Mathematical Olympiad (1998) and their solutions.

RESEARCH IN PROGRESS:

About my post-PhD research: in 1999, I started to work in 3-dimensional topology, see [10–12]; for the preprint [11], I was awarded in 2001 the Prize of Moscow Mathematical Society.

Now I work on lower bounds for the Matveev complexity of 3-manifolds. By definition, the complexity $c(M^3)$ is the minimal possible number of vertices of an almost special spine of M^3. The following properties of $c(M^3)$ were established by S. V. Matveev: (i) $c(M) = c(M_1) + c(M_2)$ whenever M is the connected sum of M_1 and M_2; (ii) for any k, there exist only finitely many prime 3-manifolds M such that $c(M) < k$. No lower bounds for$c(M^3)$ were known until 2001 (see a paper by Matveev and Pervova and [12]), except for those obtained by a computer enumeration of all 3-manifolds up to complexity 9.

In the preprint [10], "natural" spines of the lens spaces and genus one Stallings manifolds are constructed.

These days I am trying to prove twoconjectures.

Conjecture 1. *Let M^3 be the total space of T^2 bundle over S^1 with the monodromy operator $A \in SL(2,\mathbb{Z})$. Then $c(M^3) \geq d(A) + 5$, where $d(A)$ is the number defined in* [7].

The preprint [11] is devoted to the analysis of this conjecture.

For the case of punctured torus bundle $M_n \to S^1$ with the monodromy $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^n$, it is shown in [12] that $c(M_n) = 2n$. This is the first calculation of complexity for an infinite series of 3-manifolds.

Conjecture 2. *For coprime positive integers (p,q), by $E(p,q)$ denote the number of subtractions that the Euclid algorithm uses to convert the pair (p,q) into $(1,0)$. Then $c(L(p,q)) = E(p,q) - 3$, where$L(p,q)$ stands for a lens space.*

Conjectures 1 and 2 mean that the spines constructed in [10] are minimal (i.e., contain the minimal possible number of vertices). Conjecture 2 implies, in particular, that $E(p,a) = E(p,b)$ providedthat $a < p$, $b < p$, and $ab = 1 \mod p$. Indeed, we have $L(p,a) = L(p,b)$; for an algebraic proof, see [11].

RESEARCH PLAN:

Conjecture 1 above can be reduced to the following statement. Let Δ be a triangulation of T^2 and $A\Delta$ be its image under monodromy operator A. Suppose that $\widetilde{\Delta}$ is a triangulation of the cylinder $T^2 \times [0,1]$ inducing the triangulation Δ, respectively, $A\Delta$, of the torus $T^2 \times \{0\}$, respectively, $T^2 \times \{1\}$. Then $\widetilde{\Delta}$ contains at least $c(A)$ tetrahedra. (This statement is very similar to the one contained in "Remarks and questions" section of the paper "Rotation distance, triangulations, and hyperbolic geometry" by Sleator, Tarjan, and Thurston; it is also related to the notion of Gromov norm, but only integer coefficients are now allowed in the 3-chains.) It seems realistic that the proof of the last statement can be found.

Note that $SL(2,\mathbb{Z})$ is a subgroup of the group $SL(2,\mathbb{R})$, which acts on the hyperbolic plane H^2 by isometries. One can canonically assign a point $x \in H^2$ to a graph $K = F \cap P$, where $F = T^2$ is a fiber and P is a spine of $M(A)$. Roughly speaking, this point represents the coordinate system on T^2 such that the "twisting" of K with respect to this coordinate system is as small as possible. The conjecture above follows (up to a factor $\sqrt{3}/2$) from another conjecture: a flip in K shifts the point $x(K)$ at some distance d; then the maximum value of d is attained if and only if K contains only one θ-curve, see [11].

The class $\mathcal{M}$ of 3-manifolds fibered by tori over S^1 is relatively small but important: every manifold with Sol or Nil geometry either belongs to the class $\mathcal{M}$ or has a double covering in this class.

It would be interesting (and it should be possible) to apply methods of [11] to hyperbolic 3-manifolds fibered over S^1.

Below are some other topics of research.

Questions related to convex curves. After results of the paper [6], I have a strong feeling that in many respects all convex curves (that is, all Chebyshev systems of functions, or all disconjugate differential equations on the circle with scalar monodromy operator) have the same properties. For example, recently it was proved independently by V. I. Arnol'd and B. Shapiro that the number of connected components of the space of trigonometric polynomials of degree n having $2n$ different critical values is equal to the number of up-down cyclically ordered sequences consisting of $2n$ pairwise different numbers, that is, to the Euler–Bernoulli number E_n. Geometrically, this means that any connected component of the space of functions on S^1 with $2n$ pairwise different critical values contains as a representative some height function on the standard trigonometrical convex curve in $\mathbb{R}^{2n}$ defined by the mapping $\varphi \mapsto (\sin\varphi, \cos\varphi, \sin 2\varphi, \dots, \cos n\varphi)$. I want to prove that this is the case for arbitrary convex curve in $\mathbb{R}^{2n}$.

Other problems here are related to locally convex curves, that is, the curves with no flattening points. What are the homotopy and homology groups of the space of closed locally convex curves in $\mathbb{R}\mathbf{P}^n$?

Questions related to wave fronts. It is very interesting to understand what a complex wave front is. In particular, I'm looking for a complex version of the following situation. Consider a cooriented surface in $\mathbb{R}\mathbf{P}^n$ with the standard metric induced from the unit sphere, which is the total space of 2-fold covering. Let a wave front starts at this surface and moves with unit velocity. Then the front obtained at the moment $t = \pi/2$ is nothing but the cooriented surface projectively dual to the original one, and we have a "continuous model" of the projective duality. In the complex situation, we should consider, probably, the sphere $S^{2n+1} \subset \mathbb{C}^{n+1}$, which is the total space of an obvious S^1-bundle over $\mathbb{C}\mathbf{P}^n$. The simplest possible case of the projective duality involution of $\mathbb{C}\mathbf{P}^1$ is quite instructive. I hope also that there are some connections between the geometry of this phenomena and the linear algebra underlying Seiber–Witten theory.

Finally, it is a part of my research philosophy to dig into new areas every 2–3 years. Now I plan to devote my work to tight contact structures, taut foliations, laminations on 3-manifolds etc.—the recent progress in this field is encouraging.