Algebraic Geometry

program of a start up course

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SUPPOSED PRE-KNOWLEDGE: standard basic notions of algebra (*linear and multilinear algebra, polynomials, commutative rings and ideals*) and analysis (*tangent vectors, differential forms*) should be familiar; a little practice in combinatorial topology (*topological spaces, manifolds, cell or simplicial complexes and their (co)homologies*) would be desirable but not necessary.

1. **Projective space:** ▷Homogeneous coordinates. ▷Affine covering. ▷Projective subspaces. ▷Linear projections. ▷Homogeneous forms and projective varieties. ▷Space of hypersurfaces. ▷Examples of projective morphisms: Segre and Veronese.

2. **Quadrics:** ▷Tangent space and singularities. ▷Complex projective quadrics. ▷Linear subspaces on quadrics. ▷Affine quadrics. ▷Real quadrics. ▷The space of quadrics, families of quadrics. ▷Loci of singular quadrics.

3. **Grassmanians:** ▷Affine covering. ▷Cell decomposition. ▷$G(2,4)$ and line configurations in $\mathbb{P}_3$. ▷Elements of Schubert calculus: $G(2,4)$-example and some remarks for general case. ▷Plücker's embedding.

4. **Polynomial ideals & finitely generated ring extensions:** ▷Generating ideals: Noether's rings and Hilbert's theorem. ▷Integer elements in ring extensions. ▷Examples: Gauss - Dedekind type lemmas. ▷Algebraicity and divisibility, Hilbert's Nullstellensatz. ▷Homogeneous ideals and projective varieties. ▷Elimination of variables and resultants. ▷Transcendence degree and dimension.

5. **Plane algebraic curves:** ▷Irreducible components. ▷Point multiplicities. ▷Theory of invariants and geometry: polars, Hessian, e.t.c. ▷Intersection multiplicities and Bezoult's theorem. ▷Singularities, Puizo series, blow-ups. ▷Correspondences and Chasles - Cayley - Brill formula. ▷Dual curve, Plücker's formulas.

6. **Algebraic manifolds:** ▷General concept of manifold: standard cards and structure sheaf. ▷Affine algebraic variety: its points and coordinate ring. ▷Maximal spectrum and Zarissky topology. ▷Structure sheaf and algebraic morphisms. ▷Prime spectrum and schemes. ▷Geometrical properties of ring homomorphisms. ▷Finite morphisms, dimension.

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7. **Tangent spaces and differentials:** ▷Cotangent space at a point, conormal sheaf. ▷Tangent vectors and derivations. ▷Example: Lie algebras of classical algebraic groups. ▷Tangent bundles for Grassmannians, Euler's exact sequence. ▷Singular points, tangent cone. ▷Blow up.

8. **Some advanced examples (depending on a time):** ▷27 straight lines on a cubic surface. ▷Plane quartics and quintics. ▷Quadrics containing a canonical curve. ▷Fermat & Luroth...

If the time allows:

9. **Vector bundles:** ▷Locally trivial bundles via transition functions. ▷A sheaf of sections. ▷Čech's H^0 and H^1 cohomologies. ▷Vector bundles on $\mathbb{P}_1$ (via Gauss diagonalization method). ▷Line bundles, divisors and Picard group. ▷Linear systems. ▷Line bundles on projective spaces. ▷Line bundles and morphisms into projective spaces.

10. **Towards cohomologies:** ▷Resolving modules via generators and relations, exact sequences and syzigies. ▷Modules over rings of principal ideals. ▷Koszyl and de Rham complexes. ▷Hilbert's syzigies theorem. ▷A little about splitting, Tor's, Ext's, and cohomological dimension. ▷Sheaves of modules and ideals, their morphisms and exact sequences. ▷Coherent sheaves. ▷Sketch cohomologies of sheaves. ▷Example: cohomologies $H^i(\mathbb{P}_n\,;\ \Lambda^k\Omega\otimes\mathcal{O}(n))$.

Recommended textbooks:

[C]: C. H. Clemens *A scrapbook of complex curve theory*. Plenum Press.

[H]: J. Harris *Algebraic Geometry. A first course*. Springer Verlag.

[R]: M. Ried *Undergraduate algebraic geometry*. LMS Student Texts 12, CUP.

[S]: I. R. Shafarevich *Basic algebraic geometry*. Vol. 1, 2. Springer Verlag.