

Exercises to Lecture VIII

- VIII.1.** Suppose that one surface is covered by another surface. What is the relation between their Euler characteristics, if the covering is n -fold?
- VIII.2.** Prove that the sphere with g_1 handles can be covered by the sphere with g_2 handles ($g_1, g_2 \geq 2$) iff $g_1 - 1$ is a divisor of $g_2 - 1$.
- VIII.3.** Construct a nonregular covering of the wedge product of two circles.
- VIII.4.** Construct two regular coverings of the wedge product of two circles that are not homotopy equivalent to each other.
- VIII.5.** Prove that for any $n \geq 2$ the wedge product of two circles can be covered by the wedge product of n circles.
- VIII.6.** Prove that if the base surface of a covering $p: N^2 \rightarrow M^2$ is orientable, then so is the covering surface N^2 .
- VIII.7.** Let the covering surface N^2 of a covering $p: N^2 \rightarrow M^2$ is orientable. Is it true that the base surface M^2 is orientable?
- VIII.8.** Can $\mathbb{R}P^2$ cover the sphere?
- VIII.9.** Can the torus T^2 cover T^2 by a 3-fold covering?
- VIII.10.** Can $\mathbb{R}P^2$ be covered by the plane?