III.1. Prove that $D^n/\partial D^n \approx S^n$.

III.2. Prove that the space $S^1 \times S^1$ is homeomorphic to the space obtained by the following identication of points of the square $0 \le x,\ y \le 1$ belonging to its sides: $(x, 0) \sim (x, 1)$ and $(0, y) \sim (1, y)$. (This space is called the *torus*.)

III.3. Prove that the following spaces are homeomorphic:
(a) the set of lines in $\mathbb{R}^{n+1}$ passing through the origin;
(b) the sphere S^n with identified diametrically opposite points (every pair of diametrically opposite points is identified);
(c) the disc D^n with diametrically opposite points of the boundary sphere $S^{n-1} = \partial D^n$ identified.

III.4. Prove that the following spaces are homeomorphic:
(a) the set of complex lines in $\mathbb{C}^{n+1}$ passing through the origin;
(b) the sphere $S^{2n+1} \subset \mathbb{C}^{n+1}$ with identified points of the form λx for every $\lambda \in \mathbb{C}$, $|\lambda| = 1$ (for any fixed point $x \in S^{2n+1}$);
(c) the disc $D^{2n} \subset \mathbb{C}^n$ with points of the boundary sphere $S^{2n-1} = \partial D^{2n}$ of the form λx for every $\lambda \in \mathbb{C}$, $|\lambda| = 1$ identified (for any fixed point $x \in S^{2n-1}$).

III.5. Prove that $CD^n \approx D^{n+1}$ and $\Sigma D^n \approx D^{n+1}$.

III.6. Prove that $CS^n \approx D^{n+1}$ and $\Sigma S^n \approx S^{n+1}$.

III.7. Prove that $\mathbb{R}P^1 \approx S^1$ and $\mathbb{C}P^1 \approx S^2$.

III.8. Prove that $S^n * S^m \approx S^{n+m+1}$.

III.9. Prove that $\mathbb{R}^n \setminus \mathbb{R}^k \approx S^{n-k-1} \times \mathbb{R}^{k+1}$, где $\mathbb{R}^k \subset \mathbb{R}^n$ is the set $\{(x_1, \dots, x_k, 0, \dots, 0)\}$.

III.10. Prove that $S^{n+m-1} \setminus S^{n-1} \approx \mathbb{R}^n \times S^{m-1}$ where $S^{n-1} \subset S^{n+m-1}$ is standard: $S^{n+m-1} = \{(x_1, \dots, x_{n+m}) \mid x_1^2 + \dots + x_{n+m}^2 = 1\}$ and $S^{n-1} = \{(x_1, \dots, x_n, 0, \dots, 0) \mid x_1^2 + \dots + x_n^2 = 1\}$.

III.11. Prove that $(S^p \times S^q)/(S^p \vee S^q) \approx S^{p+q}$.

III.12. Prove that $T^2 \# \mathbb{R}P^2 \approx 3\mathbb{R}P^2$.

III.13. (a) Prove that Kl#Kl is homeomorphic to the Klein bottle with one handle attached.
(b) Prove that $\mathbb{R}P^2 \#$Kl is homeomorphic to the projective plane with one handle attached.

III.14. Prove that if a surface M_1 is nonorientable, then for any surface M_2 the surface $M_1 \# M_2$ is nonorientable.

III.15. (a) Prove that the two surfaces-with-holes obtained from the same closed triangulated surface by removing two different open 2-simplices from it are homeomorphic.
(b) Show that the connected sum of surfaces is well defined.

III.16. Let $I = [0, 1]$. Prove that the space $S^1 \times I$ is not homeomorphic to the Möbius band.