1. Compute the Hochschild homology and cohomology of A (with coefficients in A) for the algebras

   (a) The ground field $\Bbbk$.

   (b) "Dual numbers" $A = \Bbbk[\varepsilon]/\langle\varepsilon^2 = 0\rangle$.

   (c) The tensor algebra $T(V)$ of a vector space V.

   (d) Weyl algebra, i.e. the algebra of polynomial differential operators in n variables (begin with $n = 1$).

2. Prove, that if A is unital, one can compute the Hochschild (co)homology of A using the *normalized* Hochschild complex: $\overline{C}_n(A, M) = M \otimes \overline{A}^{\otimes n}$, where $\overline{A} = A/\Bbbk\cdot 1$ (the differential in $\overline{C}_*(A, M)$ is induced from the usual Hochschild complex).

3. Prove, that if $A = A' \times A''$ (as algebras), then $HH_*(A) = HH_*(A') \oplus HH_*(A'')$.

4. Prove, that the antisymmetrization map $\sigma : M \otimes \wedge^n A \to M \otimes A^{\otimes n}$ induces a morphism of complexes from the Chevalley complex of $A, [,]$ with coefficients in M (i.e. A, viewed as Lie algebra with respect to the usual commutator) and the hochschild complex of A with coefficients in M.

5. Prove that the first Hochschild homology of any algebra A is isomorphic to the bimodule of "universal Kähler differentials", i.e. to the bimodule $\Omega(A)$, spanned by the elements $a\,db,\ a, b \in A$, where $d : A \to \Omega(A)$ is the map, verifying generalised Leibniz rule:

$$d(ab) = da\,b + a\,db.$$

6. Observe, that the multiplication of M on either side by the elements of the center of A induces the structure of $Z(A)$-bimodule on the complex $C_*(A, M)$ (and $C^*(A, M)$). Prove that on the level of (co)homology both left- and right-module structures coincide.

7. Prove that if $B = Mat_n(A)$, then the map $Tr : B \otimes B^{\otimes n} \to A \otimes A^{\otimes n}$, given by the formula

$$Tr(B^0 \otimes B^1 \otimes \cdots \otimes B^n) = \sum_{i_0, i_1, \dots, i_n} B^0_{i_0 i_1} \otimes B^1_{i_1 i_2} \otimes \cdots \otimes B^n_{i_n i_0}$$

   induces a chain map between the corresponding Hochschild complexes. Find a similar map for the Hochschild cochain complexes[1].

8. * Prove that for any $U\mathfrak{g}$-bimodule M, the following is true: $H_*(U\mathfrak{g}, M) = H_*(\mathfrak{g}, M^{ad})$, where on the right stand the Chevalley-Eilenberg homology and M^{ad} is the $\mathfrak{g}$-complex, obtained from $U\mathfrak{g}$-bimodule by the formula $x \cdot_{ad} m = xm - mx$.

[1] In effect, one can show that there is an isomorphism of (co)homology $HH_*(A) \cong HH_*(B)$ in this case; this statement is called *the Morita equivalence* property of Hochschild (co)homology