"Young Russian Mathematics" award
Scientific report for 2016
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1 Scientific Results and Papers

Among the dynamical systems, the set of hyperbolic systems (also known as Axiom A) is the one of best understood. Though they exhibit so-called "deterministic chaos" and are exponentially hard for numerical analysis, they are also known to have nice statistical properties somewhat close to sequences of independent identically distributed random variables. Very next to hyperbolic systems, but already on a frontier of the unknown, lies the set of partially hyperbolic dynamical systems.

In collaboration with J. De Simoi, C. Liverani, C. Poquet, we studied a simple class of fast-slow partially hyperbolic dynamical systems and showed that the (properly rescaled) behavior of the slow variable is very close to a Freidlin–Wentzell type random system for times that are rather long, but much shorter than the metastability scale. This allows for sharper results than the previous local limit theorems and large deviation type theorems proved for the same class of dynamical systems in [1], [2].

This new results for medium time scales allowed, in turn, for understanding the statistical properties of these dynamical systems for arbitrary long times. In particular, we could find the loci for physical measures and find explicit formulas for their Lyapunov exponents.

One particularly interesting consequence of this is that our systems can (generically) exhibit a "sink" with all the Lyapunov exponents positive. This phenomenon seems to be very counter-intuitive at a first glance but turns out to be related to the lack of absolutely continuity of the central foliation. Hereby we provided an explicit construction for a natural class of dynamical foliations which do not admit Fubini-type theorems.

The aforementioned results are published in paper [3].

2 New Projects and Collaborations

During this year, my scientific interests extended from pure theoretical dynamical systems theory and ergodic theory to applications of mathematics in neuroscience.

With Vadim Nikulin (Charité University Medicine Berlin, HSE Moscow), we started a new project in which we aim to create a new method for finding synchronized activity in human brain cortex based electro- and magnetoencephalography recordings. Vadim Nikulin is an established scientist in electrophysiology, while I bring in my knowledge in probability theory, signal processing, and optimization theory.

I also started collaborating with Boris Gutkin (ENS Paris, HSE Moscow) and his student Nikita Novikov (HSE) on a project joint between dynamical systems theory and neuroscience. The goal is to design new realistic mathematical models of human memory, which we later analyze from the dynamical systems theory viewpoint.

3 Participation in Schools and Conferences

In February, I participated in a scientific school "Modern biology and future biotechnology" which tool place near Moscow. My specific interest in this particular edition of the school was the emphasis on neuroscience.

In June, I participated in 2nd International Conference on Mathematical NeuroScience (ICMNS 2016) and the associated workshop held in Juan Les Pins, France.

In September, I presented a poster on a conference "Cognition, Computation, Communication and Perception" held in Higher School of Economics, Moscow. The poster was related to my emerging project with Vadim Nikulin on mathematics of brain rhythms.

In November, I participated in a workshop "Active and passive methods of the brain mapping" in Higher School of Economics, Moscow.

4 Teaching

I am currently a postdoc at Centre for Cognition and Decision Making, Higher School of Economics, Moscow. This is an international research unit closely related to the Higher School of Economics's Department of Psychology.

In Fall 2016 I taught a few lectures for the course "Computational Neuroscience" on Master's programme "Cognitive Sciences and Technologies: From Neuron to Cognition". I also organized and taught a tutorial seminar "Programming in MATLAB" for bachelors of the Department of Psychology.

References

[1] De Simoi J, Liverani C. Fast-slow partially hyperbolic systems: beyond averaging. Part I (Limit Theorems). arXiv preprint arXiv:1408.5453. 2014 Aug 23.

[2] De Simoi J, Liverani C. Statistical properties of mostly contracting fast-slow partially hyperbolic systems. Inventiones mathematicae. 2014 Aug:1-81.

[3] de Simoi J, Liverani C, Poquet C, Volk D. Fast–Slow Partially Hyperbolic Systems Versus Freidlin–Wentzell Random Systems. Journal of Statistical Physics. 2016 Jul 14:1-30.