REPORT ON THE CONTEST "YOUNG RUSSIAN MATHEMATICS" (2018)

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Results 2018

(1) Every textbook on Lie algebras includes the statement known as Poincaré-Birkhoff-Witt Theorem:

The associated graded to the PBW filtration on the universal enveloping algebra $U(\mathfrak{g})$ is isomorphic to the symmetric algebra $S(\mathfrak{g})$

Given a symmetric operad $\mathcal{P}$ and a $\mathcal{P}$-algebra V, the universal enveloping algebra $\mathsf{U}_{\mathcal{P}}$ is an associative algebra whose category of modules is isomorphic to the abelian category of V-modules. We study the notion of PBW property for universal enveloping algebras over a given operad $\mathcal{P}$. In case $\mathcal{P}$ is Koszul a criterion for PBW property is found. Necessary condition on Hilbert series for $\mathcal{P}$ is found. Moreover, given any symmetric operad $\mathcal{P}$ together with a Gröbner basis G, a condition is given on the structure of the underlying trees associated with leading monomials of G sufficient for the PBW property to hold. In particular, we proved that universal enveloping algebras for the operad of *PreLie algebras*, the operad of *pairs of compatible Lie brackets* and the operad of *Zinbiel algebras* satisfy PBW property and the operad of *Poisson algebras*, the operad of *permutative algebras* and the operad of *Leibnitz algebras* do not satisfy PBW.

(2) We prove that the celebrated operad controlling pre-Lie algebras is, from many different viewpoints, very rigid from the point of view of homotopy theory. In particular, we show that the deformation complexes of each of the following maps of operads is almost acyclic:

$$\mathsf{Lie} \hookrightarrow \mathsf{PreLie} \overset{Id}{\to} \mathsf{PreLie} \to \mathsf{Assoc}$$

Moreover, the same methods can be used to show that the operadic twisting of the operad of PreLie algebras is quasiisomorphic to the operad of Lie algebras. The latter explains why one deals with the structure of a Lie algebra on a deformation complex of an operad.

Papers 2018

(1) "PBW property for Universal enveloping algebras over an operad"
 preprint available at math.arXiv:1807.05873

(2) "Homotopical rigidity of the Pre-Lie operad" (with V. Dotsenko)
 in preparation

Scientific conferences and seminar talks (2018)

(1) Topology seminar at Stockholm University: "Compactified moduli spaces of rational curves with marked points as homotopy quotients of operads"

(2) Talks in Mathematical Physics (ETH Zurich): "Cacti groups, real locus of Deligne-Mumford compactification $\bar{M}_{0,n+1}$.
(3) Topology and Geometry seminar (Hebrew University): "Compactified moduli spaces of rational curves with marked points as the homotopy quotient of an operad"
(4) "Characteristic classes and Intersection theory" seminar at NRU HSE organized by M.Kazarian-S.Lando: "On real moduli spaces of stable rational curves"

Teaching (2018)

(1) Algebraic geometry, NRU Higher School of Economics, IV year bachelor students and master students, fall 2018, 2 hours seminars per week, lectures are given by Vadim Vologodsky.

Program include
- Definitions and basic properties of schemes;
- Quasicoherent and coherent sheaves;
- Pickard group and divisors.

(2) Basic algebra for I year bachelor students, NRU Higher School of Economics, fall 2018, 4 hours exercises class per week + tests (once per 3 week). (lectures are presented by Valery Gritsenko. I was the course coordinator.)

This course was separated into two parts: basic algebraic structures and basics of linear algebra. Parts of the algebraic structures include:
- Field, simple extension, quotient rings;
- Euclidean domains, Chinese remainder theorem;
- Complex numbers, Gaussian integers;
- Group actions, orbits and stabilizers;

Parts of the linear algebra in the course include:
- System of linear equations and Gauss method;
- Matrix algebra and group of linear transformations;
- Determinants;

(3) I am currently a scientific supervisor at NRU HSE of
- 3 bachelor students of third grade,
- 2 bachelor student of first grade,
- 2 master student,
- 1 bachelor diploma thesis was written this year under my supervision.