

MINI-COURSE PROPOSED BY FRANÇOIS LAUDENBACH:

OPEN BOOKS AND TWISTED OPEN BOOKS. APPLICATION TO  
FOLIATIONS AND CONTACT STRUCTURES IN DIMENSION 3

The concept of open book decomposition goes back to H. Winkelnkemper (1973) and, according to old works by J.W. Alexander in the twenties, every closed orientable 3-manifold admits such a decomposition. In his 1991 paper, E. Giroux gave the material for a Morse theoretical approach to open book decompositions. As a consequence, every closed 3-manifold, orientable or not, has such a decomposition.

In an appropriate sense, an open book decomposition of the 3-manifold  $M$  carries a codimension-one foliation and also, when  $M$  is orientable, a contact structure (Thurston–Winkelnkemper, 1975). But, these foliations and contact structures are co-orientable, since the normal bundle to an open book (which makes sense despite the binding set) must be trivial. Ten years ago, E. Giroux showed that every co-orientable contact structure on  $M^3$  is carried by some open book.

For overcoming the triviality of the normal bundle, in a joint work with G. Meigniez, we introduced the concept of *twisted* open book and we proved a result, similar to the one of Giroux, for concordance classes of codimension-one foliations with singularities ( $\Gamma_1$ -structures of Haefliger); here the singularities which are meant are singularities of functions:

*Every  $\Gamma_1$ -structure on  $M^3$ , whose normal bundle embeds into the tangent bundle  $\tau M$ , is concordant to a non-singular foliation carried by a (twisted) open book.*

This statement is the 3-dimensional part of a famous theorem due to W. Thurston (1976).

In this course, this topic will be discussed in a rather detailed manner. The starting point will be a dynamical approach to twisted open books by considering suitable *pseudo-gradients* vector fields of Morse  $\Gamma_1$ -structures. That it will lead to a simplified proof of Giroux's theorem is still a hope.