Topics in explicit algebraic geometry: course proposal

Maxim Leyenson

Program

This program might take more than one semester to realize.

Topics in algebraic surfaces

This section contains some explicit examples of algebraic surfaces (and their geometry) which were not studied in the previous courses.

- Cubic surfaces in $\mathbb{P}^3$
  - Linear system of plane cubic curves through 6 points on $\mathbb{P}^2$: brief reminder
  - 27 lines on a cubic surface: proof.
  - Picard group of a cubic surface.
  - Relation to the root system E_6 and the Weyl group E_6
- Other del Pezzo surfaces.
- Examples of birational isomorphisms of surfaces. Cremona transformations of the plane.
- Ruled surfaces. Elementary transformations.
- Elliptic surfaces: examples. Degenerate fibers.
- Abelian surfaces: examples.
- K3 surfaces, introduction.
  - Space quartics, and the notion of K3-surfaces.
  - Classical definition of K3 surfaces.
  - Examples of K3 surfaces of genus 2, 3, 4 and 5.

    - ⋆ Kummer K3 surfaces.
- ◦ Some surfaces of general type
    - ⋆ Surface geography, short introduction.
    - ⋆ Barlow surface.
    - ⋆ Some other examples.
    - ⋆ Noether's line.
    - ⋆ Hirzebruch's examples of surfaces with $c_1^2 = 3c_2$.
- ◦ Some non-classical surfaces in positive characteristics.
- ◦ Introduction to surface singularities.
    - ⋆ Resolution of isolated singularities of surfaces; some examples.
    - ⋆ Du Val singularities and their resolution.

Grassmannian varieties: topics

- ◦ Plucker quadric.
- ◦ Intersection theory on Gr(2,4)

Topics on curves: moduli spaces

- ◦ Curves of genus 5: detailed study, and trisecant lines. Plane models.
- ◦ Severi varieties of plane curves. Dimension of the moduli space of curves: algebraic computation.
- ◦ Moduli space of curves: Idea of the construction; and genus two case.

Topics on curves: space curves

- ◦ Space curves - geography. Halphen's results.
- ◦ Castelnuovo's results.

Topics on curves: vector bundles

- ◦ Vector bundles on curves. Moduli of vector bundles – idea of the construction; and an example: vector bundles on genus 2 curves.

Introduction into the quadratic complex of lines in $\mathbb{P}^3$

- Linear complex of lines
- Quadratic complex of lines: definition
- Associated Kummer surface.
- Associated K3 surface.
- Fano variety of X is an abelian surface.